

7 Ratio and Proportion

7.1 Equivalent Ratios

Orange squash is to be mixed with water in a ratio of 1 : 6; this means that for every unit of orange squash, 6 units of water will be used. The table gives some examples:

<i>Amount of Orange Squash</i> (cm ³)	<i>Amount of Water</i> (cm ³)
1	6
20	120
5	30

The ratios 1 : 6 and 20 : 120 and 5 : 30 are all equivalent ratios, but 1 : 6 is the *simplest* form.

Ratios can be simplified by dividing both sides by the same number: note the similarity to fractions. An alternative method for some purposes, is to reduce to the form 1 : n or n : 1 by dividing *both* numbers by either the left-hand-side (LHS) or the right-hand-side (RHS). For example:

$$\text{the ratio } 4 : 10 \text{ may be simplified to } \frac{4}{4} : \frac{10}{4} \Rightarrow 1 : 2.5$$

$$\text{the ratio } 8 : 5 \text{ may be simplified to } \frac{8}{5} : \frac{5}{5} \Rightarrow 1.6 : 1$$



Example 1

Write each of these ratios in its simplest form:

(a) 7 : 14

(b) 15 : 25

(c) 10 : 4



Solution

(a) Divide both sides by 7, giving

$$\begin{aligned} 7 : 14 &= \frac{7}{7} : \frac{14}{7} \\ &= 1 : 2 \end{aligned}$$

(b) Divide both sides by 5, giving

$$\begin{aligned} 15 : 25 &= \frac{15}{5} : \frac{25}{5} \\ &= 3 : 5 \end{aligned}$$

(c) Divide both sides by 2, giving

$$\begin{aligned} 10 : 4 &= \frac{10}{2} : \frac{4}{2} \\ &= 5 : 2 \end{aligned}$$



Example 2

Write these ratios in the form $1 : n$.

(a) $3 : 12$

(b) $5 : 6$

(c) $10 : 42$



Solution

(a) Divide both sides by 3, giving

$$3 : 12 = 1 : 4$$

(b) Divide both sides by 5, giving

$$\begin{aligned} 5 : 6 &= 1 : \frac{6}{5} \\ &= 1 : 1.2 \end{aligned}$$

(c) Divide both sides by 10, giving

$$\begin{aligned} 10 : 42 &= 1 : \frac{42}{10} \\ &= 1 : 4.2 \end{aligned}$$



Example 3

The scale on a map is $1 : 20\,000$. What actual distance does a length of 8 cm on the map represent?



Solution

$$\begin{aligned} \text{Actual distance} &= 8 \times 20\,000 \\ &= 160\,000 \text{ cm} \\ &= 1600 \text{ m} \\ &= 1.6 \text{ km} \end{aligned}$$



Exercises

1. Write each of these ratios in its simplest form:

(a) $2 : 6$

(b) $4 : 20$

(c) $3 : 15$

(d) $6 : 2$

(e) $24 : 4$

(f) $30 : 25$

(g) $14 : 21$

(h) $15 : 60$

(i) $20 : 100$

(j) $80 : 100$

(k) $18 : 24$

(l) $22 : 77$

2. Write in the form $1 : n$, each of the following ratios:

(a) $2 : 5$

(b) $5 : 3$

(c) $10 : 35$

(d) $2 : 17$

(e) $4 : 10$

(f) $8 : 20$

(g) $6 : 9$

(h) $15 : 12$

(i) $5 : 12$

3. Write in the form $n : 1$, each of the following ratios:
- (a) $24 : 3$ (b) $4 : 5$ (c) $7 : 10$
(d) $15 : 2$ (e) $18 : 5$ (f) $6 : 5$
4. Jennifer mixes 600 ml of orange juice with 900 ml of apple juice to make a fruit drink. Write the ratio of orange juice to apple juice in its simplest form.
5. A builder mixes 10 shovels of cement with 25 shovels of sand. Write the ratio of cement to sand:
- (a) in its simplest form,
(b) in the form $1 : n$,
(c) in the form $n : 1$,
6. In a cake recipe, 300 grams of butter are mixed with 800 grams of flour. Write the ratio of butter to flour:
- (a) in its simplest form,
(b) in the form $1 : n$,
(c) in the form $n : 1$.
7. In a school there are 850 pupils and 40 teachers. Write the ratio of teachers to pupils:
- (a) in its simplest form, (b) in the form $1 : n$.
8. A map is drawn with a scale of $1 : 50\,000$. Calculate the actual distances, in km, that the following lengths on the map represent:
- (a) 2 cm (b) 9 cm (c) 30 cm.
9. A map has a scale of $1 : 200\,000$. The distance between two towns is 60 km. How far apart are the towns on the map?
10. On a map, a distance of 5 cm represents an actual distance of 15 km. Write the scale of the map in the form $1 : n$.

7.2 Direct Proportion

Direct proportion can be used to carry out calculations like the one below:

If 10 calculators cost £120,
then 1 calculator costs £12,
and 8 calculators cost £96.



Example 1

If 6 copies of a book cost £9, calculate the cost of 8 books.



Solution

If 6 copies cost £9,

$$\begin{aligned} \text{then 1 copy costs } & \text{£} \frac{9}{6} \\ & = \text{£}1.50 \end{aligned}$$

$$\begin{aligned} \text{and 8 copies cost } & \text{£}1.50 \times 8 \\ & = \text{£}12 \end{aligned}$$



Example 2

If 25 floppy discs cost £5.50, calculate the cost of 11 floppy discs.



Solution

$$\text{If 25 discs cost } \quad \text{£}5.50 = 550\text{p}$$

$$\text{then 1 disc costs } \quad \frac{550}{25} = 22\text{p}$$

$$\begin{aligned} \text{so 11 discs cost } \quad 11 \times 22\text{p} & = 242\text{p} \\ & = \text{£}2.42 \end{aligned}$$



Exercises

- If 5 tickets for a play cost £40, calculate the cost of:
 - 6 tickets
 - 9 tickets
 - 20 tickets.
- To make 3 glasses of orange squash you need 600 ml of water. How much water do you need to make:
 - 5 glasses of orange squash,
 - 7 glasses of orange squash?
- If 10 litres of petrol cost £8.20, calculate the cost of:
 - 4 litres
 - 12 litres
 - 30 litres.
- A baker uses 1800 grams of flour to make 3 loaves of bread. How much flour will he need to make:
 - 2 loaves
 - 7 loaves
 - 24 loaves?

5. Ben buys 21 football stickers for 84p. Calculate the cost of:
- (a) 7 stickers (b) 12 stickers (c) 50 stickers.
6. A 20 m length of rope costs £14.40.
- (a) Calculate the cost of 12 m of rope.
- (b) What is the cost of the rope, per metre?
7. A window cleaner charges n pence to clean each window, and for a house with 9 windows he charges £4.95.
- (a) What is n ?
- (b) Calculate the window cleaner's charge for a house with 13 windows.
8. 16 teams, each with the same number of people, enter a quiz. At the semifinal stage there are 12 people left in the competition.
- How many people entered the quiz?
9. Three identical coaches can carry a total of 162 passengers. How many passengers in total can be carried on seven of these coaches?
10. The total mass of 200 concrete blocks is 1460 kg. Calculate the mass of 900 concrete blocks.

7.3 Proportional Division

Sometimes we need to divide something in a given ratio. Malcolm and Alison share the profits from their business in the ratio 2 : 3. This means that, out of every £5 profit, Malcolm gets £2 and Alison gets £3.



Example 1

Julie and Jack run a stall at a car boot sale and take a total of £90. They share the money in the ratio 4 : 5. How much money does each receive?



Solution

As the ratio is 4 : 5, first add these numbers together to see by how many parts the £90 is to be divided.

$4 + 5 = 9$, so 9 parts are needed.

Now divide the total by 9.

$\frac{90}{9} = 10$, so each part is £10.

Julie gets 4 parts at £10, giving $4 \times £10 = £40$,

Jack gets 5 parts at £10, giving $5 \times £10 = £50$.



Example 2

Rachel, Ben and Emma are given £52. They decide to divide the money in the ratio of their ages, 10 : 9 : 7. How much does each receive?



Solution

$10 + 9 + 7 = 26$ so 26 parts are needed.

Now divide the total by 26.

$\frac{52}{26} = 2$, so each part is £2.

Rachel gets 10 parts at £2, giving $10 \times £2 = £20$

Ben gets 9 parts at £2, giving $9 \times £2 = £18$

Emma gets 7 parts at £2, giving $7 \times £2 = £14$



Exercises

- Divide £50 in the ratio 2 : 3.
 - Divide £100 in the ratio 1 : 4.
 - Divide £60 in the ratio 11 : 4.
 - Divide 80 kg in the ratio 1 : 3.
- Divide £60 in the ratio 6 : 5 : 1.
 - Divide £108 in the ratio 3 : 4 : 5.
 - Divide 30 kg in the ratio 1 : 2 : 3.
 - Divide 75 litres in the ratio 12 : 8 : 5.
- Heidi and Briony get £80 by selling their old toys at a car boot sale. They divide the money in the ratio 2 : 3. How much money do they each receive?
- In a chemistry lab, acid and water are mixed in the ratio 1 : 5. A bottle contains 216 ml of the mixture. How much acid and how much water were needed to make this amount of the mixture?
- Blue and yellow paints are mixed in the ratio 3 : 5 to produce green. How much of each of the two colours are needed to produce 40 ml of green paint?

6. Simon, Sarah and Matthew are given a total of £300. They share it in the ratio 10 : 11 : 9. How much does each receive?
7. In a fruit cocktail drink, pineapple juice, orange juice and apple juice are mixed in the ratio 7 : 5 : 4. How much of each type of juice is needed to make:
 (a) 80 ml of the cocktail, (b) 1 litre of the cocktail?
8. Blue, red and yellow paints are mixed to produce 200 ml of another colour. How much of each colour is needed if they are mixed in the ratio:
 (a) 1 : 1 : 2, (b) 3 : 3 : 2, (c) 9 : 4 : 3 ?
9. To start up a small business, it is necessary to spend £800. Paul, Margaret and Denise agree to contribute in the ratio 8 : 1 : 7. How much does each need to spend?
10. Hannah, Grace and Jordan share out 10 biscuits so that Hannah has 2, Grace has 6 and Jordan has the remainder. Later they share out 25 biscuits in the same ratio. How many does each have this time?

7.4 Linear Conversion

The ideas used in this unit can be used for converting masses, lengths and currencies.



Example 1

If £1 is worth 9 French francs, convert:

- (a) £22 to Ff, (b) 45 Ff to £, (c) 100 Ff to £.



Solution

$$\begin{aligned} \text{(a)} \quad \text{£}22 &= 22 \times 9 \\ &= 198 \text{ Ff} \end{aligned}$$

$$\text{(b)} \quad 1 \text{ Ff} = \text{£} \frac{1}{9}$$

$$\begin{aligned} \text{so } 45 \text{ Ff} &= 45 \times \frac{1}{9} \\ &= \frac{45}{9} \\ &= \text{£}5 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad 100 \text{ Ff} &= 100 \times \frac{1}{9} \\
 &= \frac{100}{9} \\
 &= \text{£}11\frac{1}{9} \\
 &= \text{£}11.11 \text{ to the nearest pence}
 \end{aligned}$$



Example 2

Use the fact that 1 foot is approximately 30 cm to convert:

- (a) 8 feet to cm, (b) 50 cm to feet, (c) 195 cm to feet.



Solution

$$\begin{aligned}
 \text{(a)} \quad 8 \text{ feet} &= 8 \times 30 \\
 &= 240 \text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad 1 \text{ cm} &= \frac{1}{30} \text{ feet} \\
 \text{so } 50 \text{ cm} &= 50 \times \frac{1}{30} \\
 &= \frac{5}{3} \\
 &= 1\frac{2}{3} \text{ feet}
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad 195 \text{ cm} &= 195 \times \frac{1}{30} \\
 &= \frac{195}{30} \\
 &= \frac{13}{2} \\
 &= 6\frac{1}{2} \text{ feet}
 \end{aligned}$$



Example 3

If £1 is worth \$1.60, convert:

- (a) £15 to dollars (b) \$8 to pounds.



Solution

$$\begin{aligned} \text{(a)} \quad \text{£}15 &= 15 \times 1.60 \\ &= \$24 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \$1 &= \text{£} \frac{1}{1.60} \\ &= \text{£} \frac{10}{16} \end{aligned}$$

$$\begin{aligned} \$8 &= 8 \times \frac{10}{16} \\ &= \frac{80}{16} \\ &= \text{£}5 \end{aligned}$$



Exercises

- If £1 is worth 9 Ff, convert:
 - £6 to Fr,
 - £100 to Ff,
 - 54 Ff to £,
 - 28 Ff to £.
- Use the fact that 1 inch is approximately 25 mm to convert:
 - 6 inches to mm,
 - 80 inches to mm,
 - 50 mm to inches,
 - 1000 mm to inches.
- Before 1971, Britain used a system of money where there were 12 pennies in a shilling and 20 shillings in a pound. Use this information to convert:
 - 100 shillings into pounds,
 - 8 shillings into pennies,
 - 132 pennies into shillings,
 - 180 pennies into shillings.
- Given that a weight of 1 lb is approximately equivalent to 450 grams, convert:
 - 5 lbs to grams,
 - 9 lb into grams,
 - 1800 grams to lb,
 - 3150 grams to lb.
- Use the fact that 1 mile is approximately the same distance as 1.6 km to convert:
 - 30 miles to km,
 - 21 miles to km,
 - 80 km to miles,
 - 200 km to miles

6. On a certain day, the exchange rate was such that £1 was worth \$1.63. Use a calculator to convert the following amounts to £, giving each answer correct to the nearest pence.
- (a) \$100 (b) \$250 (c) \$75.
7. The Japanese currency is the Yen (Y). The exchange rate gives 197 Yen for every £1. Using a calculator, convert the following amounts to pounds, giving each answer correct to the nearest pence.
- (a) 1000 Y (b) 200 Y (c) 50 000 Y.
8. A weight of 1 lb is approximately equivalent to 450 grams. There are 16 ounces in 1 lb. Give answers to the following questions correct to 1 decimal place.
- (a) Convert 14 oz to lb.
 (b) Convert 200 grams to lb.
 (c) Convert 300 grams to ounces.
9. If £1 is worth 2.8 German Marks (DM), and 1 DM is worth 2800 Italian Lira (L), use a calculator to convert:
- (a) 800 DM to £, (b) 10 000 L to DM, (c) 50 000 L to £.
10. There are 8 pints in one gallon. One gallon is equivalent to approximately 4.55 litres. Use a calculator to convert:
- (a) 12 pints to litres, (b) 20 litres to pints.
- Give your answers correct to 1 decimal place.

7.5 Inverse Proportion

Inverse proportion is when an *increase in one quantity causes a decrease in another*.

The relationship between *speed* and *time* is an example of inverse proportionality: as the speed increases, the journey time decreases, so the time for a journey can be found by dividing the distance by the speed.



Example 1

- (a) Ben rides his bike at a speed of 10 mph. How long does it take him to cycle 40 miles?
- (b) On another day he cycles the same route at a speed of 16 mph. How much time does this journey take?



Solution

$$\begin{aligned} \text{(a) Time} &= \frac{40}{10} \\ &= 4 \text{ hours} \end{aligned}$$

$$\begin{aligned} \text{(b) Time} &= \frac{40}{16} = 2\frac{1}{2} \\ &= 2\frac{1}{2} \text{ hours} \end{aligned}$$

Note: *Faster speed* \Rightarrow *shorter time*.



Example 2

Jai has to travel 280 miles. How long does it take if he travels at:

- 50 mph,
- 60 mph ?
- How much time does he save when he travels at the faster speed?



Solution

$$\begin{aligned} \text{(a) Time} &= \frac{280}{50} \\ &= 5.6 \text{ hours} \\ &= 5 \text{ hours } 36 \text{ minutes} \end{aligned}$$

$$\begin{aligned} \text{(b) Time} &= \frac{280}{60} \\ &= 4\frac{2}{3} \text{ hours} \\ &= 4 \text{ hours } 40 \text{ minutes} \end{aligned}$$

$$\begin{aligned} \text{(c) Time saved} &= 5 \text{ hours } 36 \text{ mins} - 4 \text{ hours } 40 \text{ mins} \\ &= 56 \text{ minutes} \end{aligned}$$



Example 3

In a factory, each employee can make 40 chicken pies in one hour. How long will it take:

- 6 people to make 40 pies,
- 3 people to make 240 pies,
- 10 people to make 600 pies?



Solution

- (a) 1 person makes 40 pies in 1 hour.
 6 people make 40 pies in $\frac{1}{6}$ hour (or 10 minutes).
- (b) 1 person makes 40 pies in 1 hour.
 1 person makes 240 pies in $\frac{240}{40} = 6$ hours.
 3 people make 240 pies in $\frac{6}{3} = 2$ hours.
- (a) 1 person makes 40 pies in 1 hour.
 1 person makes 600 pies in $\frac{600}{40} = 15$ hours.
 10 people make 600 pies in $\frac{15}{10} = 1\frac{1}{2}$ hours.



Exercises

- How long does it take to complete a journey of 300 miles travelling at:
 - 60 mph,
 - 50 mph,
 - 40 mph?
- Alec has to travel 420 miles. How much time does he save if he travels at 70 mph rather than 50 mph?
- Sarah has to travel 60 miles to see her boyfriend. Her dad drives at 30 mph and her uncle drives at 40 mph. How much time does she save if she travels with her uncle rather than with her dad?
- Tony usually walks to school at 3 mph. When Jennifer walks with him he walks at 4 mph. He walks 1 mile to school. How much quicker is his journey when he walks with Jennifer?
- One person can put 200 letters into envelopes in 1 hour. How long would it take for 200 letters to be put into envelopes by:
 - 4 people,
 - 6 people,
 - 10 people?

6. A person can make 20 badges in one hour using a machine. How long would it take:
 - (a) 4 people with machines to make 20 badges,
 - (b) 10 people with machines to make 300 badges,
 - (c) 12 people with machines to make 400 badges?

7. A train normally complete a 270-mile journey in $4\frac{1}{2}$ hours. How much faster would it have to travel to complete the journey in 4 hours?

8. On Monday Tom takes 15 minutes to walk one mile to school. On Tuesday he takes 20 minutes to walk the same distance. Calculate his speed in mph for each day's walk.

9. Joshua shares a 2 kg tin of sweets between himself and three friends.
 - (a) How many kg of sweets do they each receive?
 - (b) How much less would they each have received if there were four friends instead of three?

10. Nadina and her friends can each make 15 Christmas cards in one hour. How long would it take Nadina and four friends to make:
 - (a) 300 cards,
 - (b) 1000 cards?