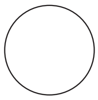
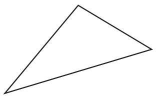
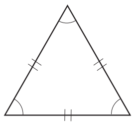
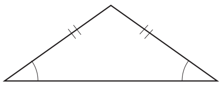
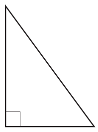
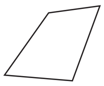
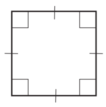
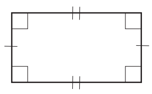
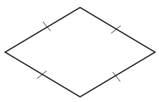
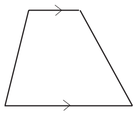
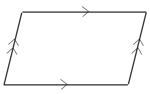
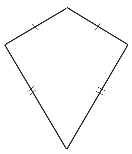
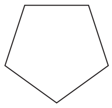
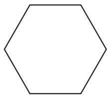
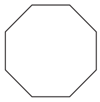
6 Nets and Surface Area

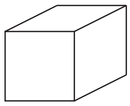
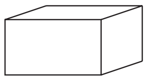
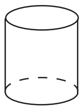
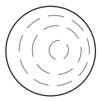
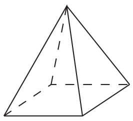
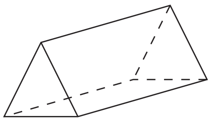
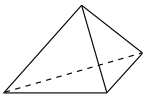
6.1 Common 2-D and 3-D Shapes

You have already met many 2-D shapes; here are some with which you should already be familiar:

NAME	*ILLUSTRATION*	*NOTES*
Circle		Symmetric about any diameter
Triangle		3 straight sides
Equilateral Triangle		3 equal sides and 3 equal angles ($= 60°$)
Isosceles Triangle		2 equal sides and 2 equal angles
Right-angled Triangle		One angle $= 90°$
Quadrilateral		4 straight sides
Square		4 equal sides and 4 right angles
Rectangle		Opposite sides equal and 4 right angles
Rhombus		4 equal sides; opposite sides parallel
Trapezium		One pair of opposite sides parallel
Parallelogram		Both pairs of opposite sides equal and parallel
Kite		Two pairs of adjacent sides equal

NAME	*ILLUSTRATION*	*NOTES*
Pentagon		5 sides (equal if *regular*)
Hexagon		6 sides (equal if *regular)*
Octagon		8 sides (equal if *regular)*

There are also several 3-D shapes with which you should be familiar:

NAME	*ILLUSTRATION*	*NOTES*
Cube		All side lengths equal (square faces), and all angles right angles
Cuboid		Faces are combination of rectangles (and squares); all angles right angles
Cylinder		Circular base
Sphere		All points on surface equidistant from centre
Pyramid (square-based)		All slant edges are equal in length in a right pyramid
Prism (triangular)		Cross-section remains the same throughout
Tetrahedron		All four faces are triangular

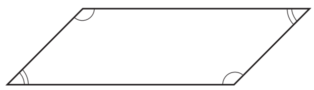
Note that a *square* is a special case of a rectangle, as it satisfies the definition; similarly, both a square and a rectangle are special cases of a parallelogram, etc.

Example 1

What is the name of the 2-D shape with 4 sides and with opposite angles equal?

Solution

The shape has to be a parallelogram.



(Note: this shape can also be a square, rhombus or rectangle as these are all special cases of a parallelogram.)

Example 2

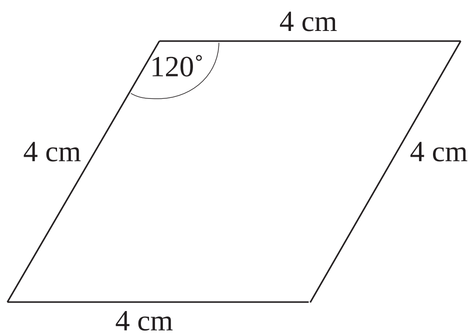
Draw accurately:

(a) a rhombus with sides of length 4 cm and one angle $120°$,

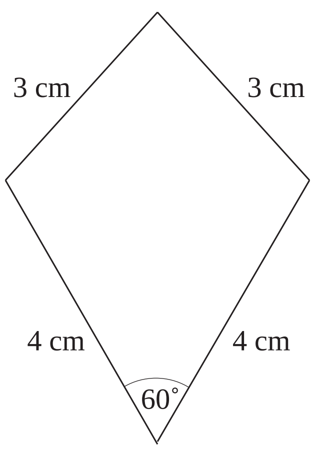
(b) a kite with sides of length 3 cm and 4 cm, and smallest angle $60°$. Measure the size of each of the other angles.

Solution

(a)



(b) Note that the smallest angle, $60°$, must be between the two longest sides.
The other angles are approximately $108°$, $108°$ and $84°$.



Exercises

1. What could be the name of the 2-dimensional shape with 4 sides, which has all angles of equal sizes?

2. What is the name of a 6-sided, 2-dimensional shape which has sides of equal lengths?

3. Draw a parallelogram with sides of lengths 3 cm and 4 cm and with smallest angle equal to $60°$.

4. Can a 4-sided, 2-dimensional shape have 4 sides of equal lengths, and *not* be a square?

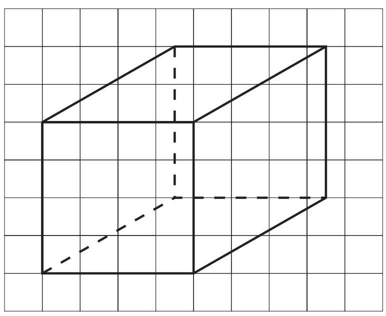
5. Can a 4-sided, 2-dimensional shape have 4 angles of equal size, and *not* be a square?

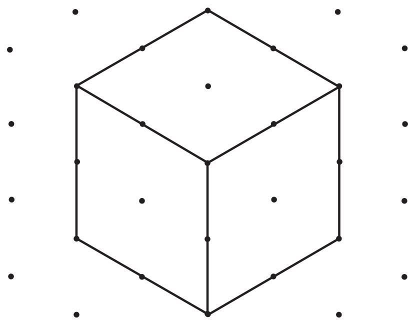
6. Name all possible 4-sided, 2-dimensional shapes that have *at least* 2 sides of equal lengths.

7. Name all possible 4-sided, 2-dimensional shapes that have *at most* 2 sides of equal lengths.

6.2 2-D Representation of 3-D Shapes

In this section we explore how to draw 3-D shapes, either on squared paper or on *isometric* (triangular spotty) paper. Examples of each for a 2 cm cube, are shown below :



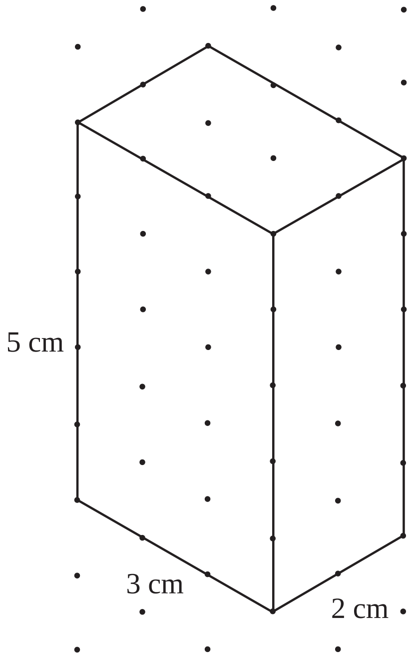


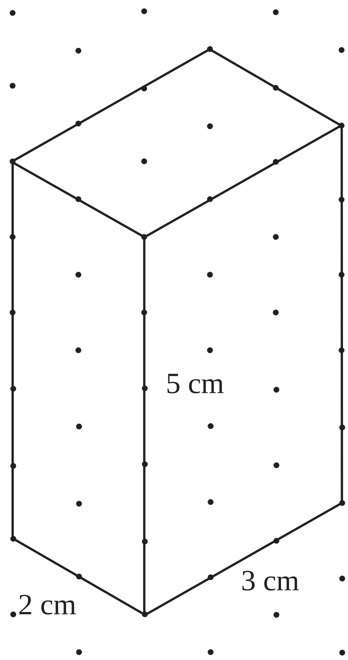
Example 1

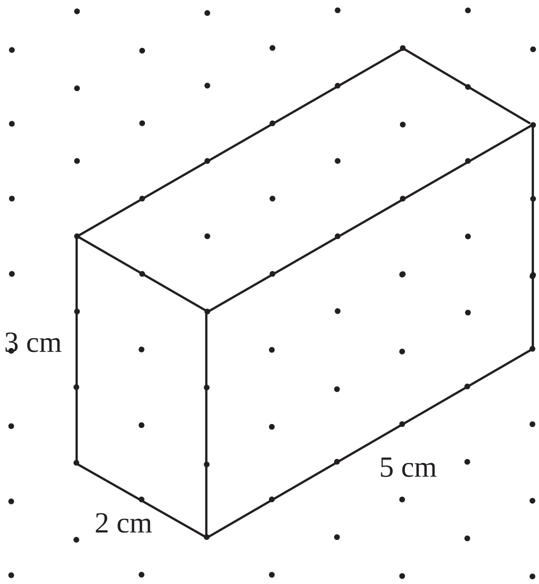
On isometric paper, draw a cuboid with sides of lengths 5 cm, 3 cm and 2 cm.

Solution

The diagrams below show three of the possible ways of drawing a $2 \text{ cm} \times 3 \text{ cm} \times 5 \text{ cm}$ cuboid.







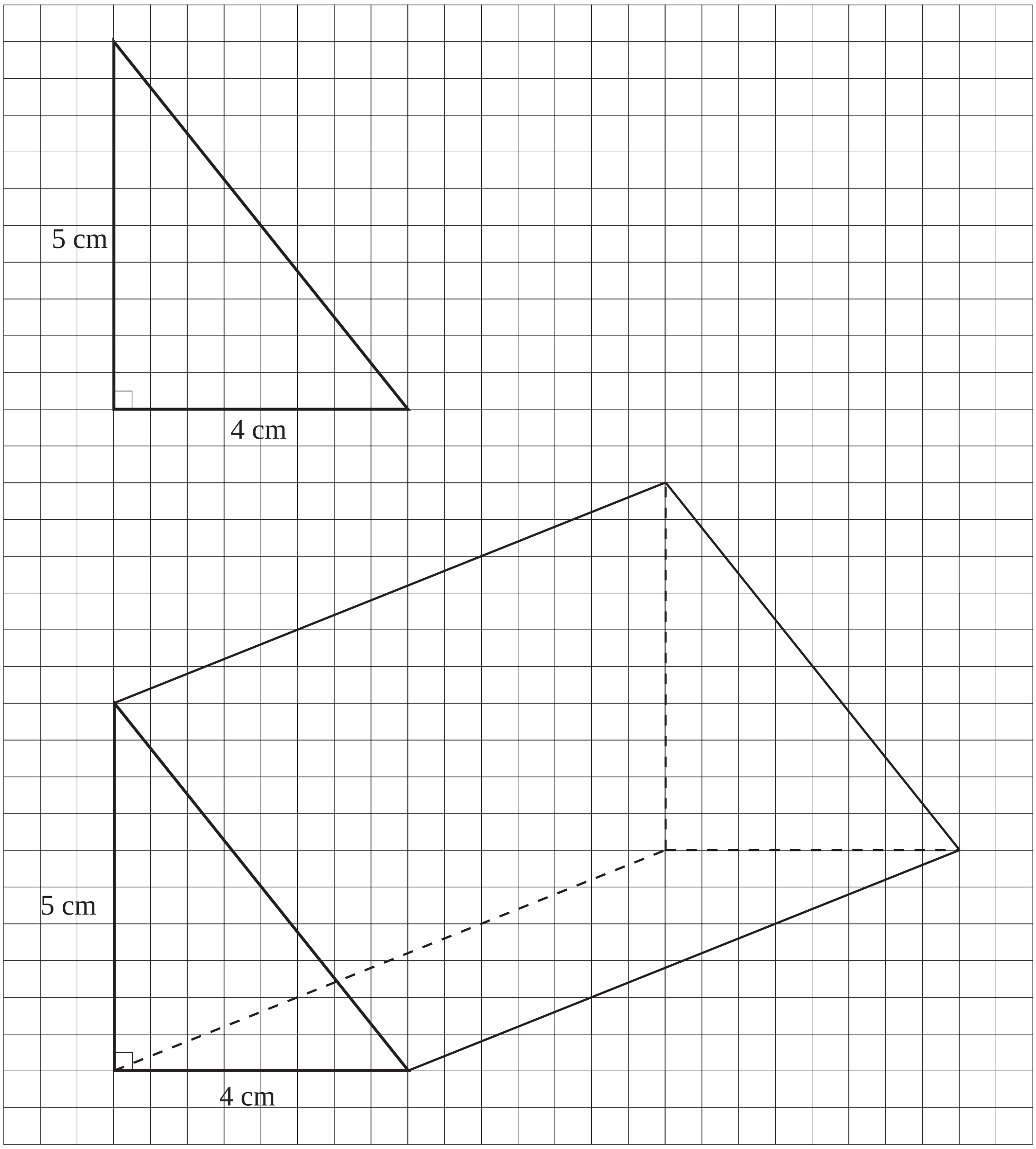
Example 2

A triangular prism has a cross-section that is a right-angled triangle with base 4 cm and height 5 cm. The length of the prism is 8 cm.

Draw the prism.

Solution

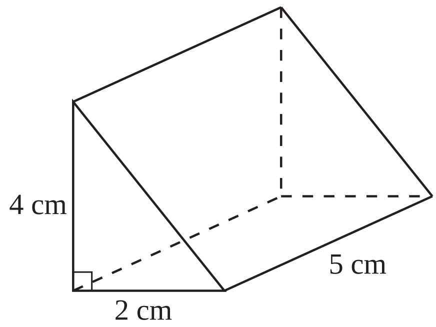
First draw the cross-section of the prism. Then draw two lines of length 8 cm, parallel to each other. Complete the triangle at the other end of the prism.



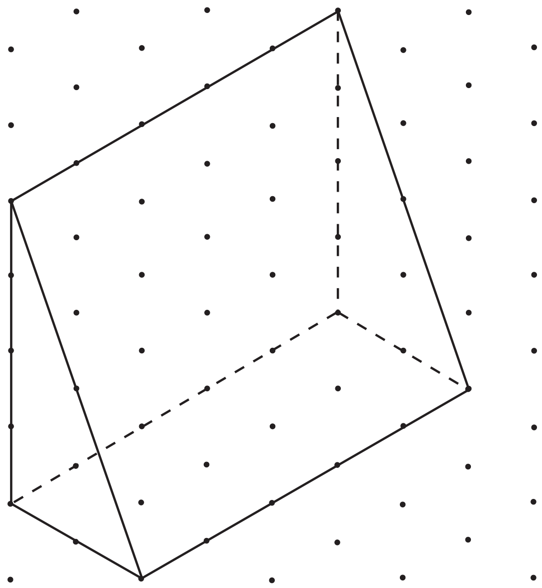
Note: Lines parallel on the object are parallel on the diagram.

Example 3

Draw this prism on isometric paper:



Solution



Exercises

(Diagrams to be drawn full size unless scale given.)

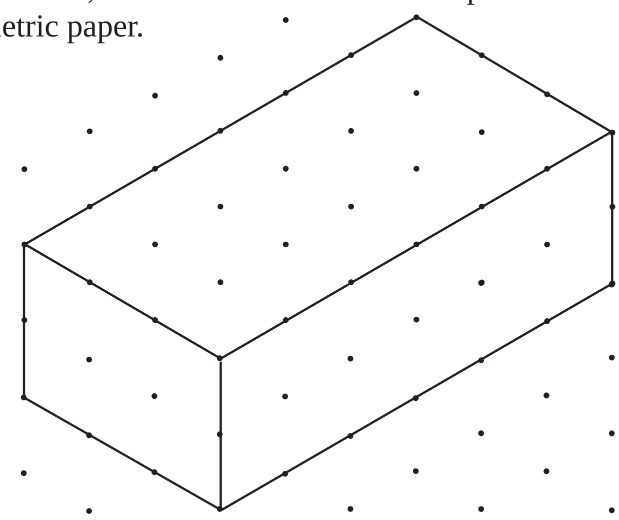
1. On isometric paper, draw a cube with sides of length 4 cm.

2. On isometric paper, draw a cuboid with sides of lengths 3 cm, 2 cm and 4 cm.

3. Three cubes with sides of length 2 cm are put side-by-side to form a cuboid. Draw this cuboid on isometric paper.

4. A cuboid has sides of lengths 3 cm, 6 cm and 2 cm. Draw three possible views of the cuboid on isometric paper.

5. The cuboid shown in the diagram opposite may be cut in half to form two triangular prisms. Draw one of these prisms on isometric paper.
   Note: *The cut may be made in three different ways.*



6. A triangular prism has a cross-section that is a right-angled triangle with base 4 cm and height 3 cm. The length of the prism is 6 cm. Draw the prism on isometric paper.

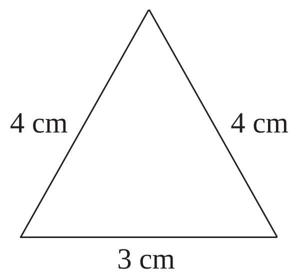
7. On plain or squared paper, draw a cube with sides of 5 cm.

8. On plain or squared paper, draw a cuboid with sides of lengths 6 cm, 4 cm and 3 cm.

9. A prism has a triangular cross-section with sides of length 6 cm. The length of the prism is 8 cm. Draw the prism on plain paper.

10. The diagram shows the cross-section of a triangular prism. The length of the prism is 5 cm.

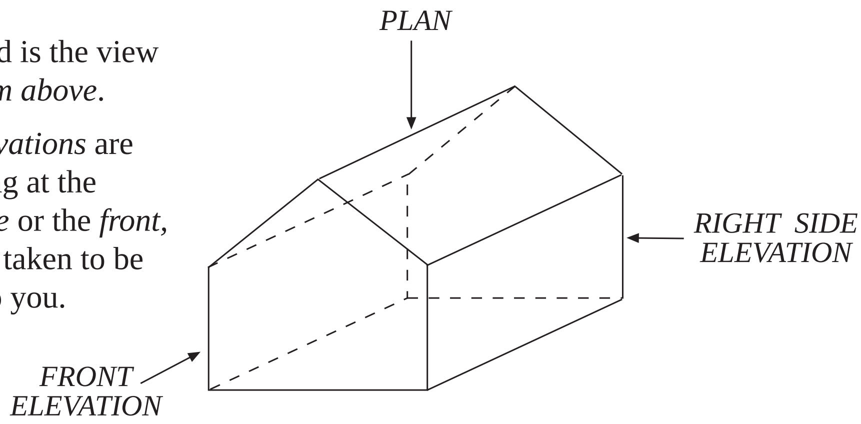
    Draw the prism on plain paper.



6.3 Plans and Elevations

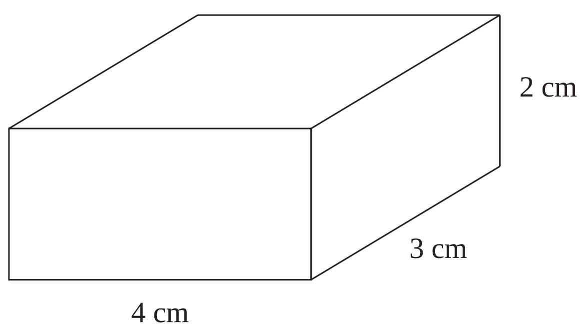
The *plan* of a solid is the view looking *down from above*.

Side and *front elevations* are drawn as if looking at the solid from the *side* or the *front*, where the front is taken to be the face nearest to you.



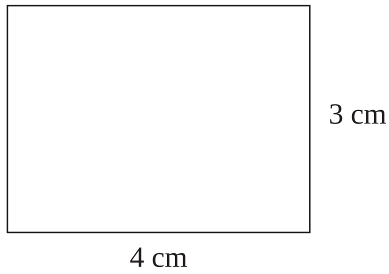
Example 1

Draw the *plan* and *elevations* of this cuboid:

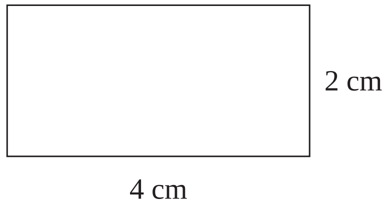


Solution

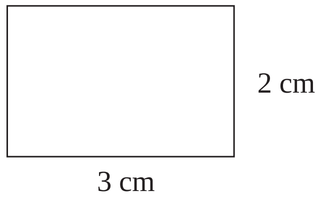
The *plan* is the view from above:



The *front elevation* is the view from the front:

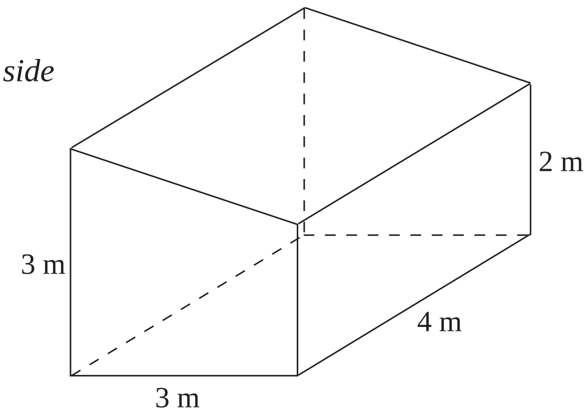


The *side elevation* is the view from the side (in this case the right and left side elevations are the same):



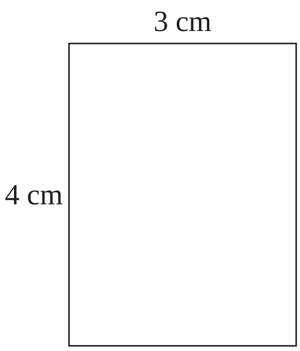
Example 2

Draw the *plan*, *front elevation* and *left side elevation* for this shed:

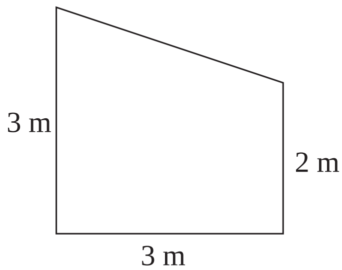


Solution

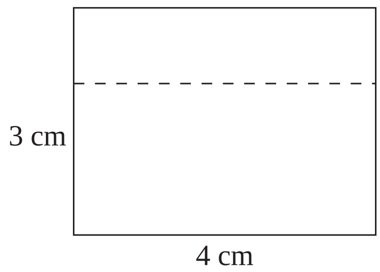
Using 1 cm for 1 m:



Plan



Front Elevation



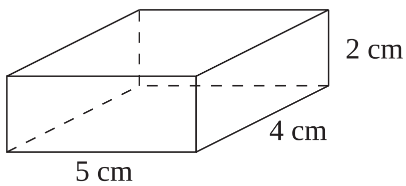
Left Side Elevation

Note: The dotted line on the left side elevation shows the position of the rear roof line which would not be visible from this viewing point.

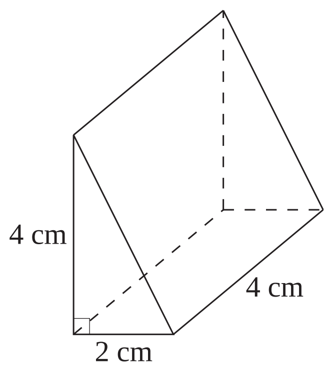
Exercises

(Diagrams to be drawn full size unless scale given.)

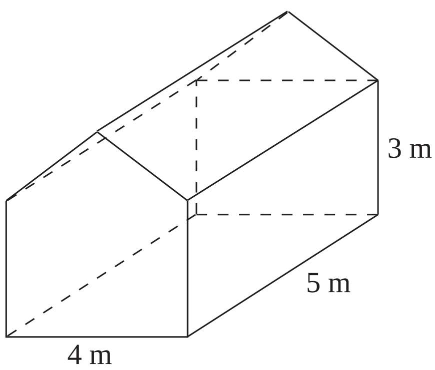
1. Draw the plan and elevations of the cuboid shown:



2. Draw the plan and elevations of the triangular prism shown:

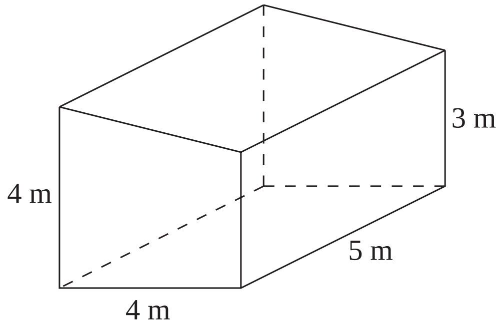


3. Draw the plan and elevations of the building shown, which is 4 m high:
   Use a scale of 1 cm to represent 1 m.

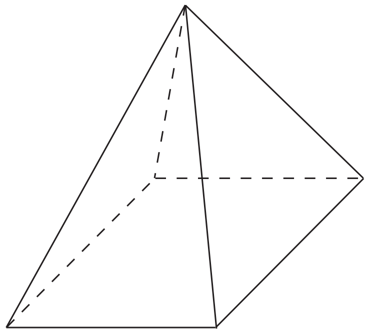


4. (a) Draw the plan and elevations of the building shown using a scale of 1 cm for 1 m:

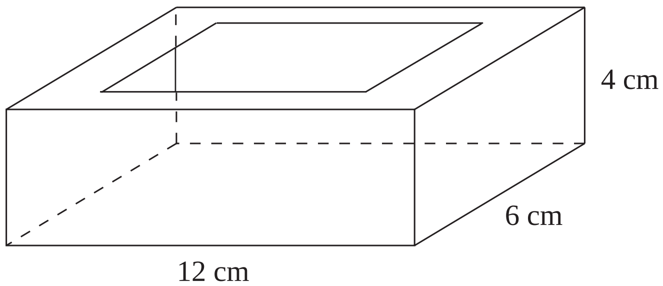
   (b) How do these views compare with those in Example 2 and in question 3 ?



5. A square-based right pyramid has a base with sides of length 4 cm. The sides of the pyramid are isosceles triangles, and the vertical height of the pyramid is 5 cm. Draw the plan, and an elevation of the pyramid.

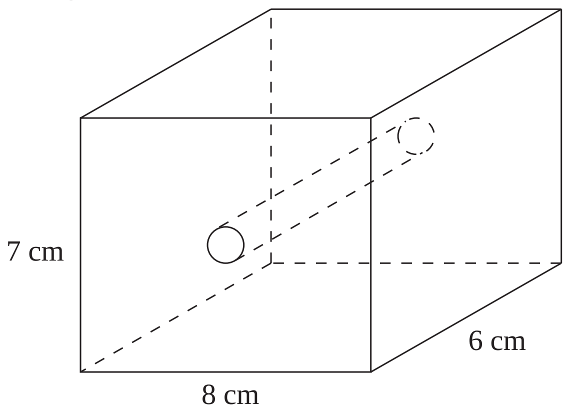


6. The diagram shows a tissue box. The opening in the centre of the top of the box is 8 cm by 4 cm.



Draw a plan and elevations of the box.

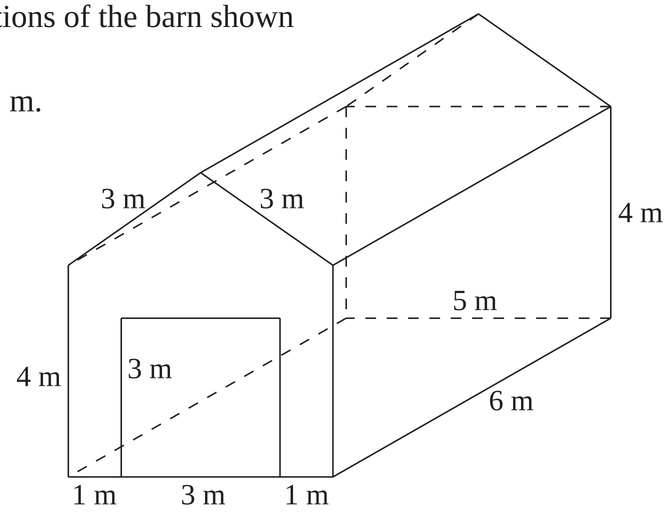
7. A hole of radius 1 cm is drilled through the middle of a block of wood as shown in the diagram:



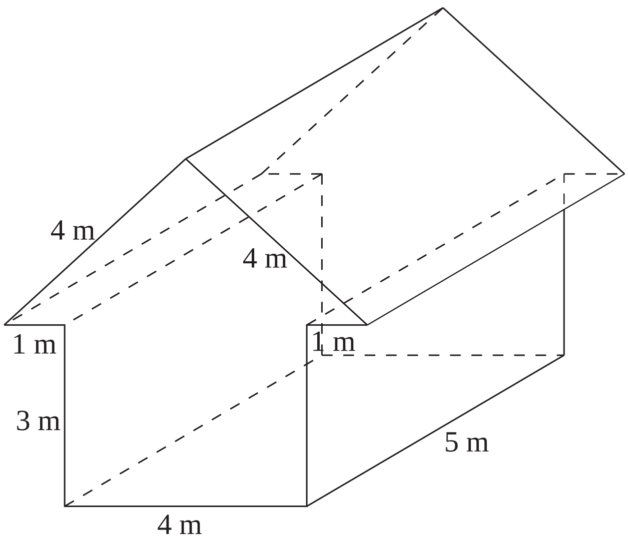
Draw the plan and elevations of the block of wood.

8. Draw the plan and elevations of the barn shown opposite:

   Use a scale of 1 cm for 1 m.



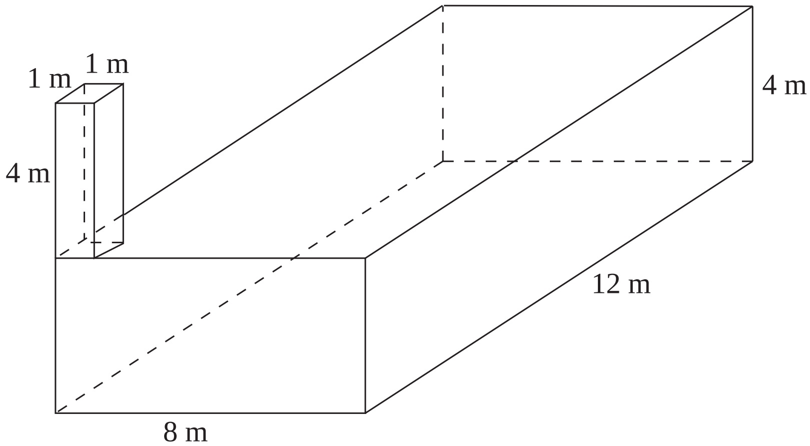
9. The sketch shows the design of a house with an overhanging roof.



   Draw the plan and elevations of the house.

10. The diagram shows a factory with a flat roof and a square-based chimney:

    Draw the plan and elevations of the building, using a scale of 1 cm for 1 m.



6.4 Nets and Surface Area of Cubes and Cuboids

A *net* can be folded up to make a solid. The diagram below shows one of the possible nets of a cube:

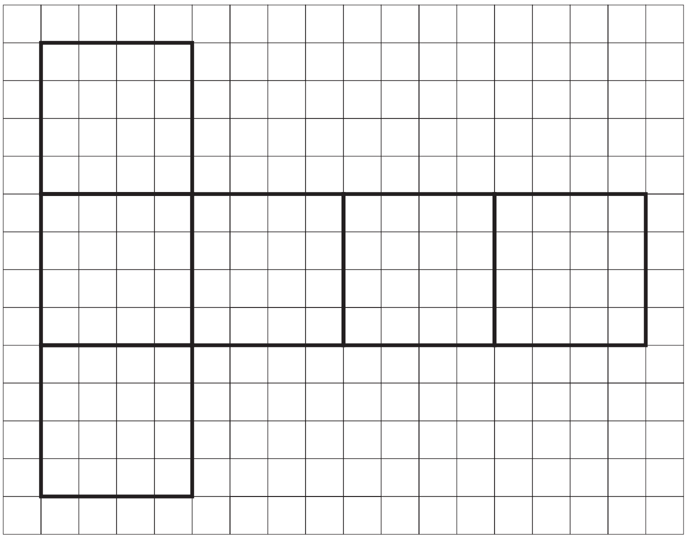
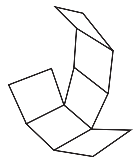
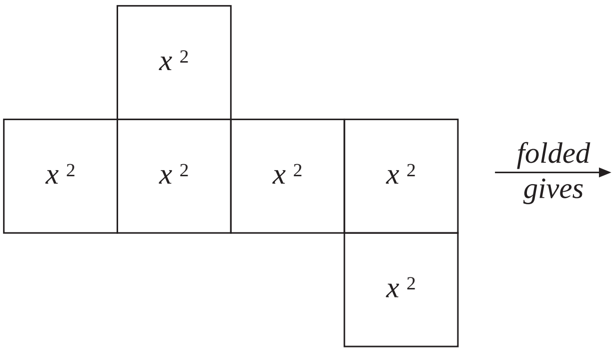


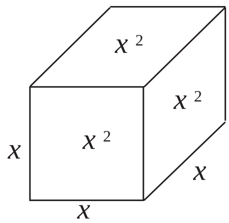
Diagram to show the net partially folded



The net of a cube is always made up of 6 squares. Each square has an area of x^2 if the length of the side of the cube is x.

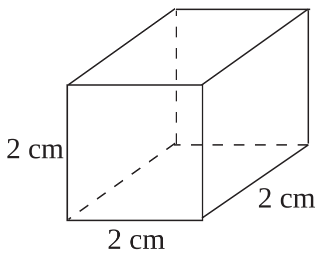
Total surface area of a cube $= 6x^2$.



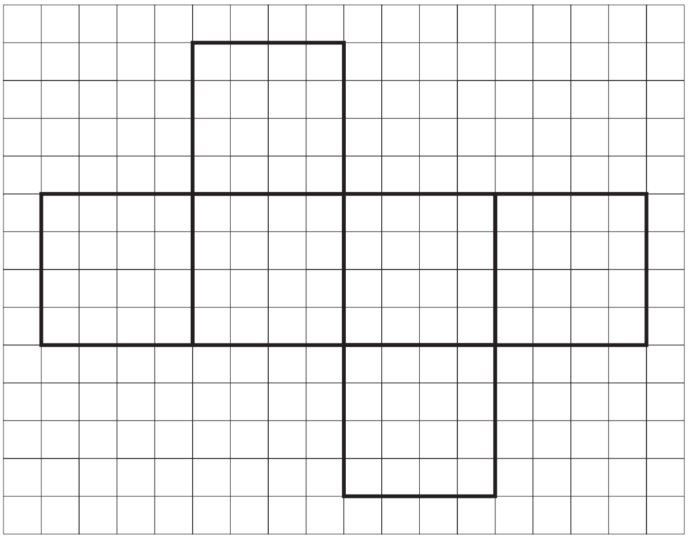


Example 1

Draw a net for the cube shown and calculate its surface area.



Solution



The net is made up of 6 squares.

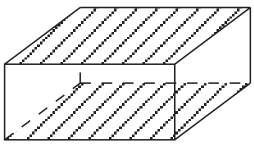
Each square has an area of 4 cm^2.

Surface area $= 6 \times 4$

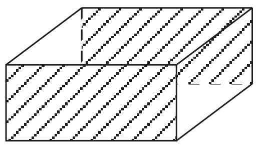
$= 24 \text{ cm}^2$.

The net of a cuboid is made up of 6 rectangles.

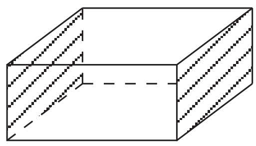
The rectangles will occur in pairs as illustrated below:



Top and bottom

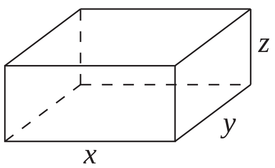


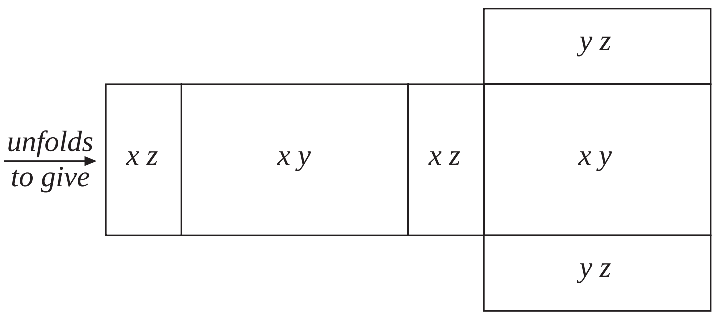
Two sides



Two ends

For this cuboid,





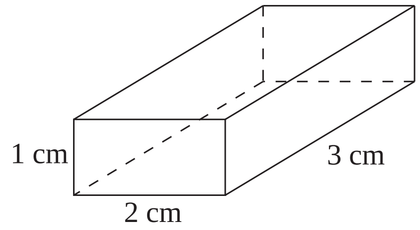
and, surface area $= xy + yz + xz + xy + yz + xz$

$= 2xy + 2yz + 2xz$

$= 2(xy + yz + xz)$

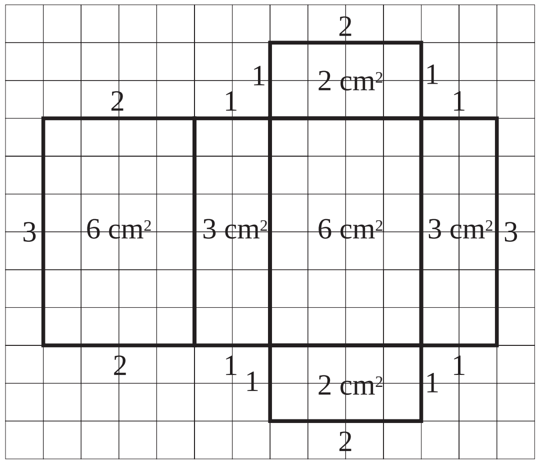
Example 2

Draw a net for the cuboid shown and calculate its surface area.



Solution

One of the possible nets for the cuboid is shown opposite, together with the area of each rectangle:



Side lengths in cm

Surface area $= 2 + 6 + 3 + 6 + 3 + 2$

$= 22 \text{ cm}^2$

You can check your solution:

$x = 2$ cm, $y = 3$ cm and $z = 1$ cm

so, using the formula $2(xy + yz + xz)$,

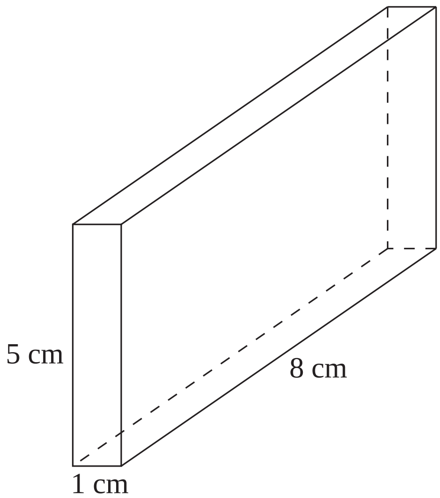
surface area $= 2(2 \times 3 + 3 \times 1 + 2 \times 1)$

$= 2 \times 11$

$= 22 \text{ cm}^2$ (as before)

Example 3

Calculate the surface area of this cuboid:



Solution

Surface area $= 2(5 \times 1 + 1 \times 8 + 5 \times 8)$

$= 2(5 + 8 + 40)$

$= 2 \times 53$

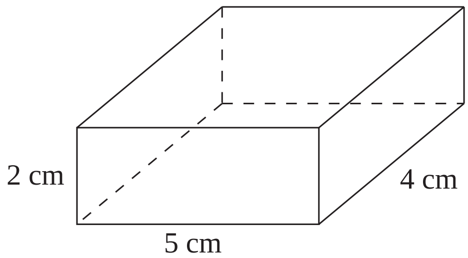
$= 106 \text{ cm}^2$

Exercises

1. Draw different arrangements of 6 squares and indicate which of them could be folded to form a cube.

2. Draw a net for a cube with sides of length 4 cm, and calculate its surface area.

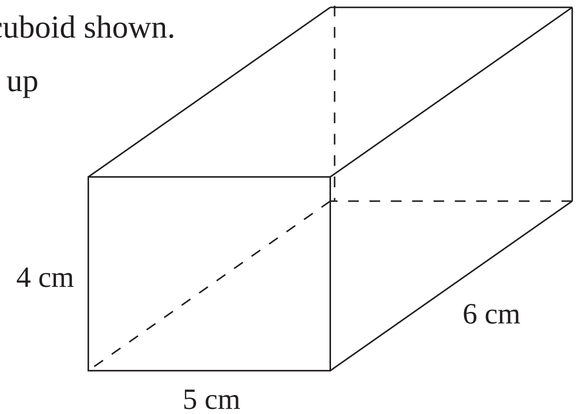
3. Draw a net for the cuboid shown, and calculate its surface area.



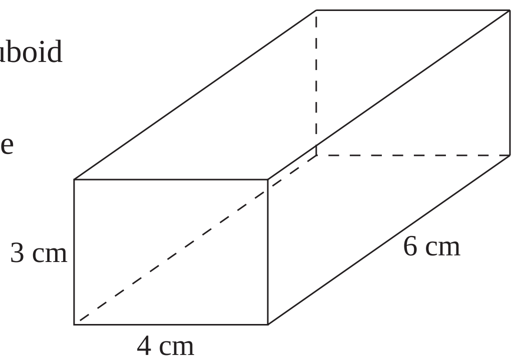
4. (a) On card, draw a net for a cube with sides of length 5 cm.
   (b) Add tabs to the net so that it can be cut out and glued together.
   (c) Cut out the net, fold it up and glue it together to make a cube.

5. Use card to make a net for the cuboid shown.

   Then add tabs, cut it out, fold it up and glue it to make the cuboid.



6. (a) Draw 2 different nets for the cuboid shown.
   (b) Calculate the surface area of the cuboid.
   (c) Do both your nets have the same surface areas?

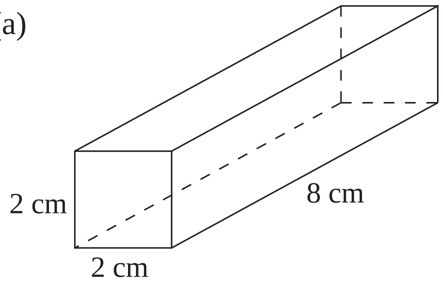


7. Without drawing a net, calculate the surface area of a cube with sides of length:

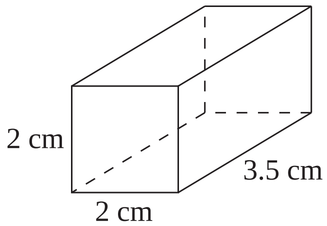
   (a) 10 cm   (b) 9 cm.

8. Calculate the surface area of each of the following cuboids:

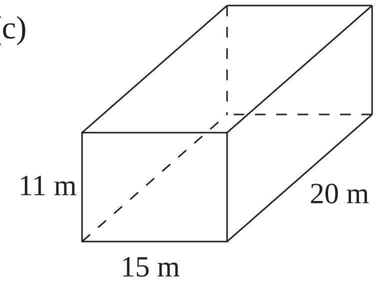
(a)



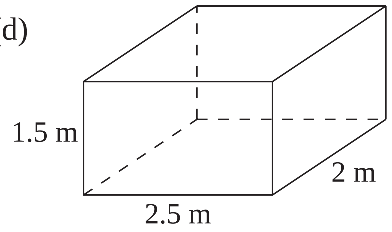
(b)



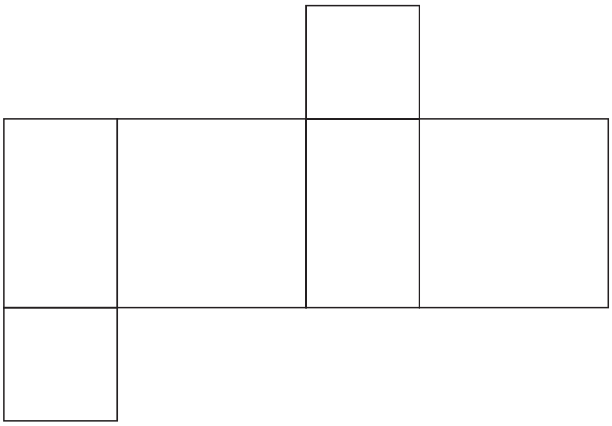
(c)



(d)



9. A diagram of a net is shown below, where two of the rectangles have been drawn inaccurately.



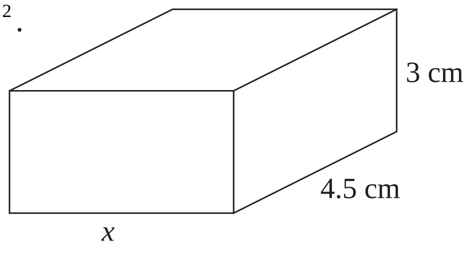
(a) Explain what is wrong with the net.

(b) Draw a modified net that would produce a cuboid, by changing two of the rectangles.

(c) Give an alternative answer to part (b).

10. The surface area of a cube is 24 cm^2. Calculate the length of the sides of the cube.

11. The surface area of this cuboid is 102 cm^2. What is the length marked x ?



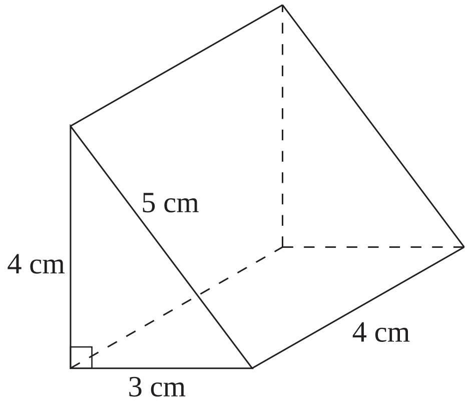
6.5 Nets of Prisms and Pyramids

In order to draw the nets of some prisms and pyramids, you will need to construct triangles as well as squares and rectangles.

Example 1

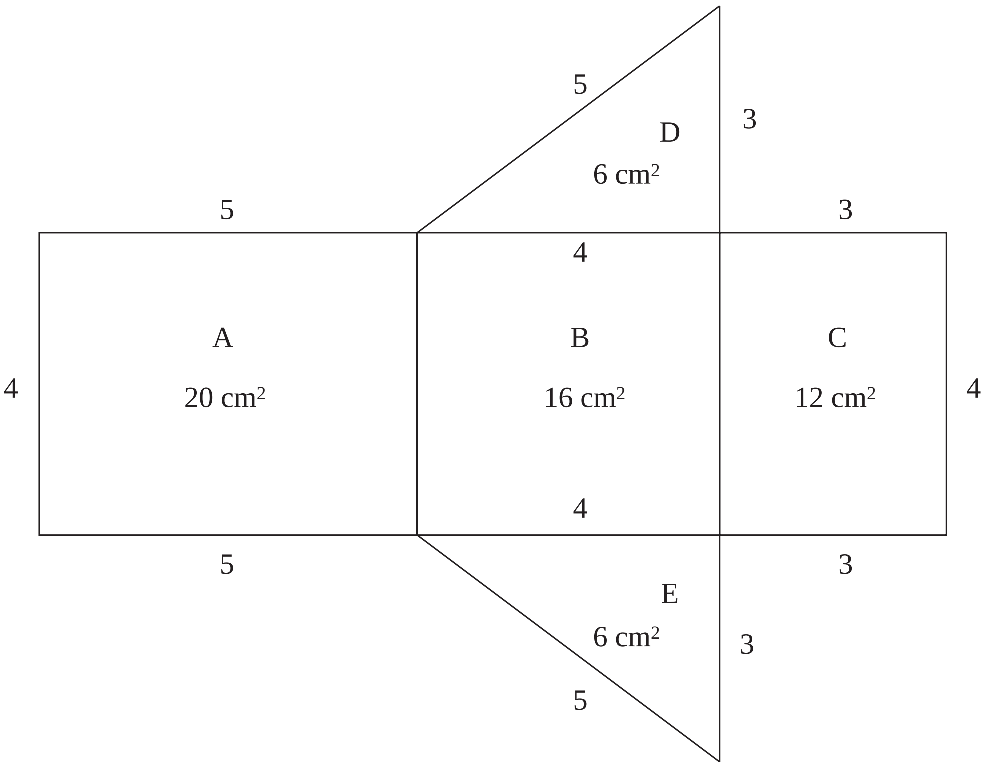
(a) Draw a net for this triangular prism:

(b) Calculate its surface area.



Solution

(a) A net is shown below where all lengths marked are in cm.



(b) The area of each part of the net has been calculated.

$$\text{Surface area} = \underset{A}{(5 \times 4)} + \underset{B}{(4 \times 4)} + \underset{C}{(4 \times 3)} + \underset{D}{\left(\frac{1}{2} \times 4 \times 3\right)} + \underset{E}{\left(\frac{1}{2} \times 4 \times 3\right)}$$

$$= 20 + 16 + 12 + 6 + 6$$

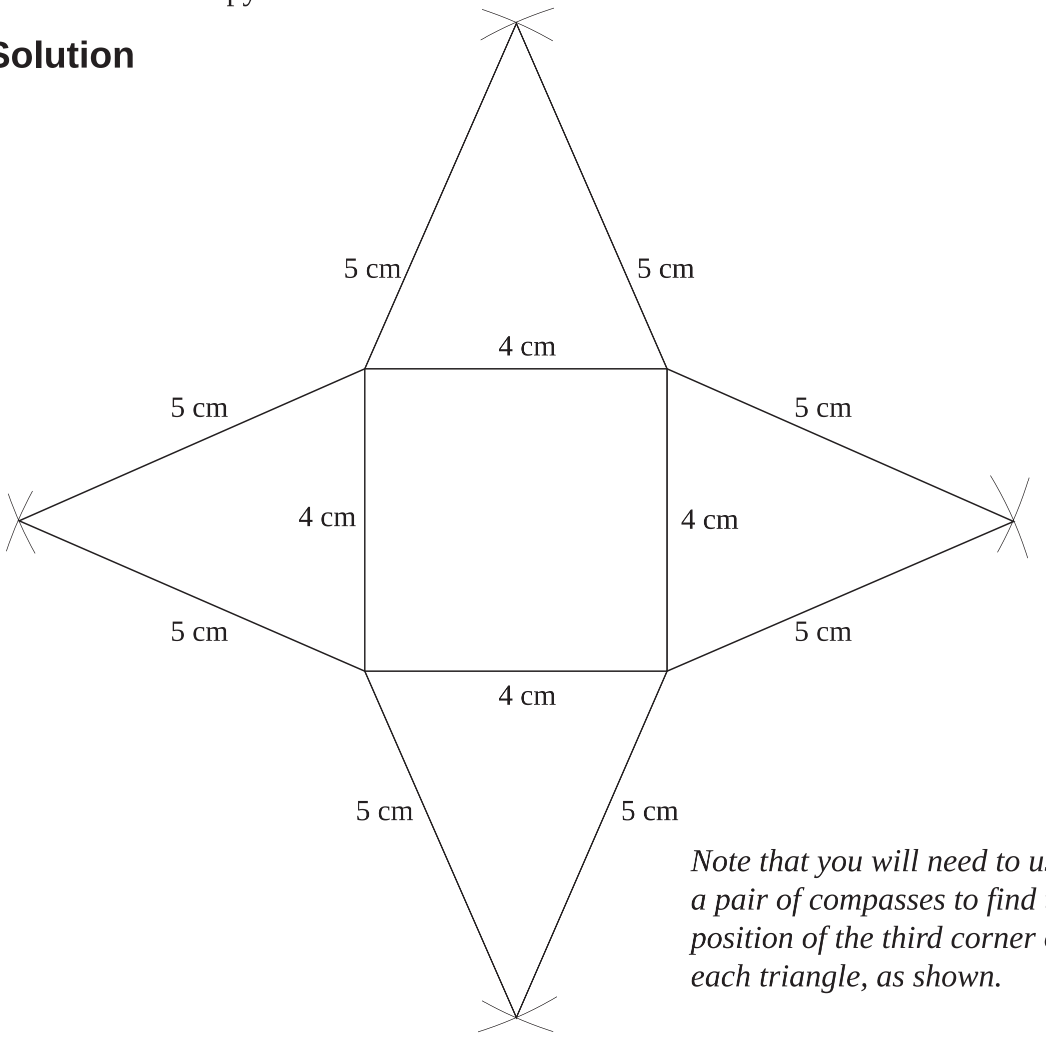
$$= 60 \text{ cm}^2$$

Example 2

The square base of a pyramid has sides of length 4 cm. The triangular faces of the pyramid are all isosceles triangles with two sides of length 5 cm.

Draw a net for the pyramid.

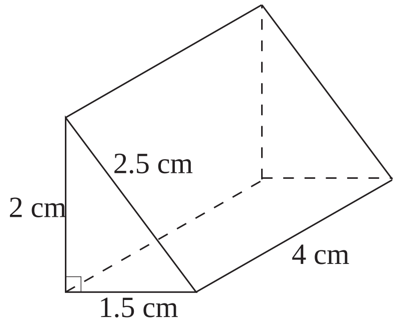
Solution



Note that you will need to use a pair of compasses to find the position of the third corner of each triangle, as shown.

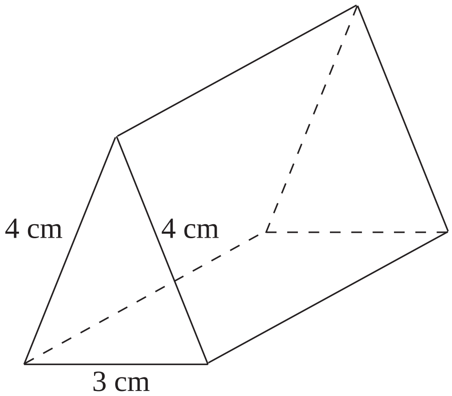
Exercises

1. Draw a net for the triangular prism shown opposite:



2. Draw a net for this prism, on card.

   Add tabs, cut it out, and then construct the actual prism.



3. A pyramid has a square base with sides of length 6 cm. The other edges of the prism have length 6 cm. Draw a net for the pyramid.

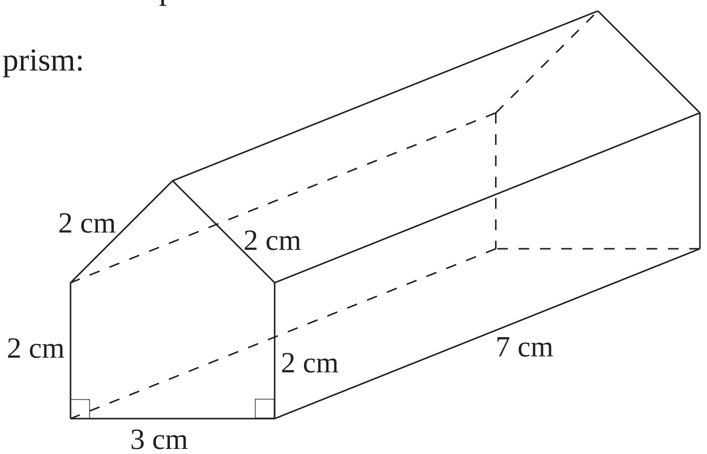
4. A pyramid has a rectangular base with sides of lengths 3 cm and 4 cm. The other edges of the pyramid have length 6 cm.

   Draw a net for this pyramid on card, cut it out and construct the pyramid.

5. A tetrahedron has four faces which are all equilateral triangles. Draw a net for a tetrahedron, which has edges of length 4 cm.

6. A square-based prism has a base with sides of length 5 cm and vertical height 6 cm. Draw the net of this prism.

7. The diagram shows a prism:

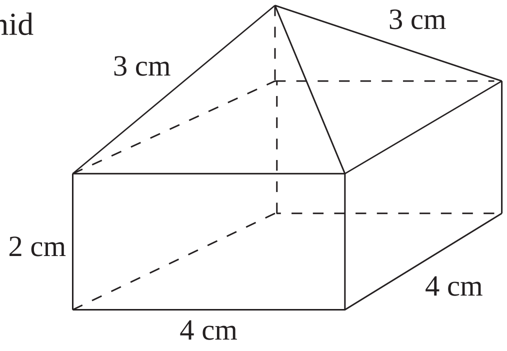


   (a) Draw a net for the prism.

   (b) Find the height of the prism.

8. A container is in the shape of a pyramid on top of a cuboid, as shown in the diagram opposite.

   Draw a net for the container.



9. The diagram below shows a square-based pyramid; the base is horizontal and AE is vertical. Draw a net for this pyramid.