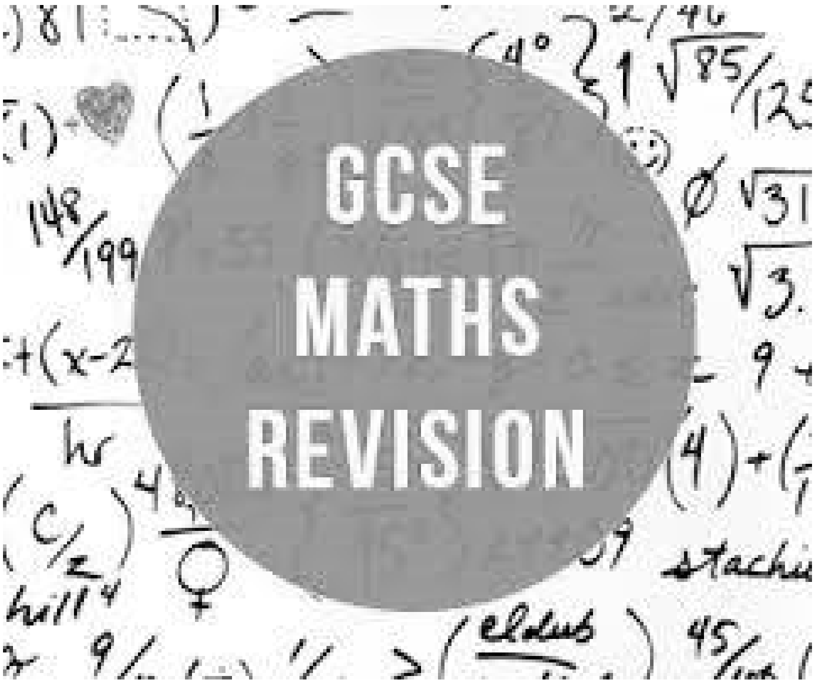
Exam Practice Workbook



Level 2

Further Maths

It's another great book from CGP...

This book is for anyone doing the **AQA Level 2 Certificate in Further Maths**.

It's overflowing with exam-style questions for every topic — and it's rounded off with two realistic practice papers to fully prepare you for your exams.

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Contents

☑ Use the tick boxes to check off the topics you've completed.

Section Five — Pythagoras and Trigonometry

Practice Papers

How to get answers for the Practice Papers
Your free Online Edition of this book includes fully worked solutions for Practice Papers 1 & 2 that you can print out. (Just flick back to the first page to find out how to get hold of your Online Edition.)

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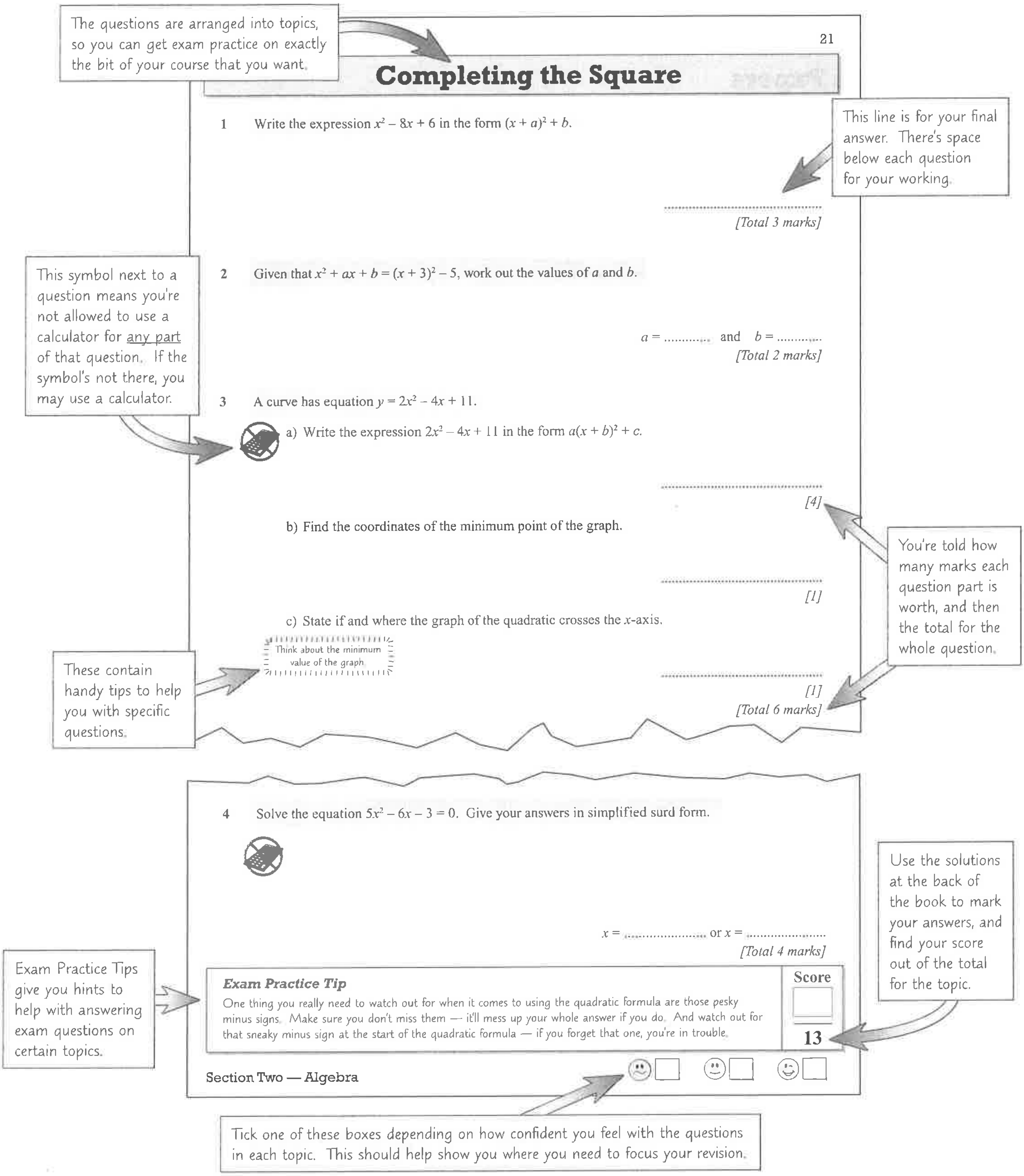
How to Use This Book

- Hold the book upright, approximately 50 cm from your face, ensuring that the text looks like this, not sıɥʇ. Alternatively, place the book on a horizontal surface (e.g. a table or desk) and sit adjacent to the book, at a distance which doesn't make the text too small to read.
- In case of emergency, press the two halves of the book together firmly in order to close.
- Before attempting to use this book, familiarise yourself with the following safety information:



The questions are arranged into topics, so you can get exam practice on exactly the bit of your course that you want.

21

Completing the Square

1 Write the expression $x^2 - 8x + 6$ in the form $(x + a)^2 + b$.

....

[Total 3 marks]

This line is for your final answer. There's space below each question for your working.

2 Given that $x^2 + ax + b = (x + 3)^2 - 5$, work out the values of a and b.

a = ... and b = ...

[Total 2 marks]

This symbol next to a question means you're not allowed to use a calculator for any part of that question. If the symbol's not there, you may use a calculator.

3 A curve has equation $y = 2x^2 - 4x + 11$.

a) Write the expression $2x^2 - 4x + 11$ in the form $a(x + b)^2 + c$.

....

[4]

b) Find the coordinates of the minimum point of the graph.

....

[1]

c) State if and where the graph of the quadratic crosses the x-axis.

Think about the minimum value of the graph.

....

[1]

[Total 6 marks]

You're told how many marks each question part is worth, and then the total for the whole question.

These contain handy tips to help you with specific questions.

4 Solve the equation $5x^2 - 6x - 3 = 0$. Give your answers in simplified surd form.

x = ... or x = ...

[Total 4 marks]

Use the solutions at the back of the book to mark your answers, and find your score out of the total for the topic.

Exam Practice Tip

One thing you really need to watch out for when it comes to using the quadratic formula are those pesky minus signs. Make sure you don't miss them — it'll mess up your whole answer if you do. And watch out for that sneaky minus sign at the start of the quadratic formula — if you forget that one, you're in trouble.

Score

13

Exam Practice Tips give you hints to help with answering exam questions on certain topics.

Section Two — Algebra

Tick one of these boxes depending on how confident you feel with the questions in each topic. This should help show you where you need to focus your revision.

Exam Tips

Exam Stuff

1) You will have two exams — one non-calculator exam and one calculator exam.
2) The non-calculator exam is 1 hr 30 mins long and worth 70 marks.
3) The calculator exam is 2 hrs long and worth 105 marks.
4) Timings in the exam are really important, so here's a quick guide...

- You should spend about a minute per mark working on each question (i.e. 2 marks = 2 mins).
- That'll leave about 15-20 minutes at the end of each exam to check back through your answers and make sure you haven't made any silly mistakes. Not to just stare at that hottie in front.
- If you're totally, hopelessly stuck on a question, just leave it and move on to the next one. You can always go back to it at the end if you've got enough time.

There are a Few Golden Rules

1) **Always, always, always make sure you read the question properly.**
   For example, if the question asks you to give your answer in metres, don't give it in centimetres.
2) **Show each step in your working.**
   You're less likely to make a mistake if you write things out in stages. And even if your final answer's wrong, you'll probably pick up some marks if the examiner can see that your method is right.
3) **Check that your answer is sensible.**
   Worked out an angle of 450° or 0.045° in a triangle? You've probably gone wrong somewhere...
4) **Make sure you give your answer to the right degree of accuracy.**
   The question might ask you to round to a certain number of significant figures or decimal places. So make sure you do just that, otherwise you'll almost certainly lose marks.
5) **Look at the number of marks a question is worth.**
   If a question's worth 2 or more marks, you're not going to get them all for just writing down the final answer — you're going to have to show your working.
6) **Write your answers as clearly as you can.**
   If the examiner can't read your answer you won't get any marks, even if it's right.

Obeying these Golden Rules will help you get as many marks as you can in the exam — but they're no use if you haven't learnt the stuff in the first place. So make sure you revise well and do as many practice questions as you can.

Using Your Calculator

1) Your calculator can make questions a lot easier for you but only if you know how to use it. Make sure you know what the different buttons do and how to use them.
2) Remember to check your calculator is in degrees mode. This is important for trigonometry questions.
3) If you're working out a big calculation on your calculator, it's best to do it in stages and use the memory to store the answers to the different parts. If you try and do it all in one go, it's too easy to mess it up.
4) If you're going to be a renegade and do a question all in one go on your calculator, use brackets so the calculator knows which bits to do first.

REMEMBER: Golden Rule number 2 still applies, even if you're using a calculator — you should still write down all the steps you are doing so the examiner can see the method you're using.

Fractions

1 Work out:

a) $4\frac{2}{5} + 3\frac{1}{4}$

....
[3]

b) $2\frac{5}{6} - 1\frac{1}{5}$

....
[3]

[Total 6 marks]

2 Give your answers to the following in their simplest form (where appropriate).

a) $4\frac{3}{5} \times 2\frac{1}{3}$

....
[2]

b) $4\frac{1}{3} \div 2\frac{3}{5}$

....
[3]

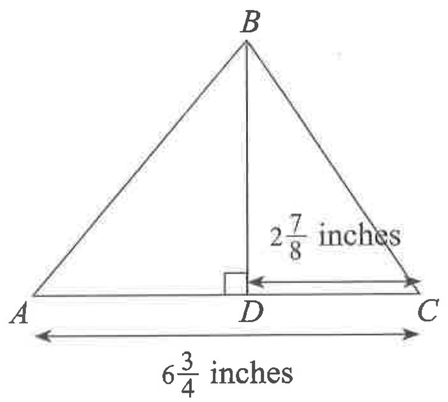
[Total 5 marks]

3 Work out the following calculation.

$7\frac{1}{5} - 2\frac{1}{4} \times 2\frac{1}{3}$

....
[Total 5 marks]

4 Given that the length of BD is two thirds the length of AC, find the area of triangle ABD below. Give your answer in its simplest form.



.... in^2
[Total 5 marks]

Score: /21

Fractions, Decimals and Percentages

1 Convert each of the following:

a) $\frac{3}{5}$ to a decimal.

.... *[1]*

b) 0.04 to a percentage.

.... *[1]*

c) 65% to a fraction in its simplest form.

.... *[1]*

[Total 3 marks]

2 Drew and Lucy did an exam and were each given a mark out of 80.

Lucy scored $\frac{56}{80}$.

a) Convert her score to a percentage.

.... % *[2]*

Drew got 90% on the exam.

b) What was Drew's mark out of 80 as a fraction?

.... *[2]*

[Total 4 marks]

3 There are 150 coloured blocks in a box. Each block is red, blue or green.

42% of the blocks are red.

$\frac{8}{25}$ of the blocks are blue.

How many of the blocks are green?

....

[Total 3 marks]

Score: ☐ / **10**

☐ ☐ ☐

Percentages

1 A bag contains round, square and triangular counters in a variety of colours.

24% of the 75 round counters are black.

$\frac{2}{5}$ of the 65 square counters are black.

18 out of the 48 triangular counters are black.

a) Which shape has the highest percentage of black counters?

....

[3]

b) What percentage of **all** the counters in the bag are black?
Give your answer to 2 d.p.

.... %

[4]

[Total 7 marks]

2 x is increased by 17% to 13 104.

What was the value of x before the increase?

....

[Total 3 marks]

3 The value of x is 125% of the value of s.
The value of y is 36% less than the value of t.

Given that $x = \frac{1}{y}$, find the value of st as a decimal.

....

[Total 4 marks]

4 Bag A contains a sweets and bag B contains b sweets. 15% of the sweets from bag A are removed and added to bag B. Both bags now contain an equal number of sweets.

Express b as a percentage of a.

....

[Total 4 marks]

Score: ___ / **18**

Ratios

1 84 is divided in the ratio 3 : 5 : 4 : 8.

How big is the largest share?

....

[Total 3 marks]

2 Simone, André, Nasir and Chloe shared £660.
Nasir got four times as much money as Chloe, Simone got twice as much money as Nasir, and André got a quarter as much money as Simone.

How much money did Simone get?

£ ....

[Total 3 marks]

3 $y = 35\,000$. Find the larger amount when $\frac{2y}{5}$ is split in the ratio 11 : 17.

....

[Total 3 marks]

4 Simplify the ratio $(4x - 3y)(2x^3 - 4y) : (y + 6x)(9x + 8y)$ as far as possible given that $x = 0$ and $y \neq x$.

.... : ....

[Total 2 marks]

5 A floor is made from red, green and white tiles. The ratio of red to green tiles in the floor is 1 : 3. The ratio of green to white tiles in the floor is 4 : 3.

Work out the fraction of tiles in the floor that are white.

....

[Total 3 marks]

6 Given that $a : b = 2 : 7$ and $b - a = 40$, work out $a + b$.

....

[Total 3 marks]

7 Two variables, f and s, are in the ratio 3 : 7. The sum of f and s is 3250.

Given that $fx + 175s = 651\ 625$, find the value of x.

....

[Total 5 marks]

8 The ratio of Georgia's age (g years) to Harry's age (h years) is 2 : 5

a) Write h in terms of g.

....

[1]

b) In 10 years' time, the ratio of their ages will be 3 : 5.
Work out Georgia's current age.

....

[4]

[Total 5 marks]

Score: $\frac{\quad}{27}$

Powers and Roots

1 **Simplify the following expressions fully.**

a) $5a^{\frac{3}{2}} \times 4a^{\frac{5}{2}}b^3$

....
[2]

b) $\dfrac{6a^{\frac{5}{2}}b^6}{3a^{\frac{1}{2}}b^4}$

....
[2]

[Total 4 marks]

2 **Answer the following:**

a) Show that $16^{\frac{3}{2}} = 64$

[2]

b) Hence, or otherwise, find the value of x if $16^x = 64^6$

$x =$ ....
[2]

[Total 4 marks]

3 **Completely simplify the expression below.**

$(4a^6)^{\frac{1}{2}} \times \dfrac{5a^3b^5}{10a^5b^2}$

....
[Total 3 marks]

4 **Write $\dfrac{x+5x^3}{\sqrt{x}}$ in the form $x^m + 5x^n$, where m and n are constants.**

....
[Total 2 marks]

Score: ___ / 13

Expanding Brackets

1 Expand the brackets in the following expressions. Simplify your answers as much as possible.

a) $4x(2x - 3)$

....

[2]

b) $5a(3a + 6ab)$

....

[2]

c) $3p^3(8 - p^2) - 4p(2p^2 - 7p)$

....

[3]

[Total 7 marks]

2 Expand and simplify $4a^2(2a - 5) + a(3a + 4a^2)$.

....

[Total 2 marks]

3 Expand the brackets in the following expressions. Simplify your answers as much as possible.

a) $(4t - 3)(2t + 5)$

....

[3]

b) $(2x + 9)^2$

....

[3]

[Total 6 marks]

4 Show clearly that the area of the triangle below can be written as $x^2 + 2x - 3$.

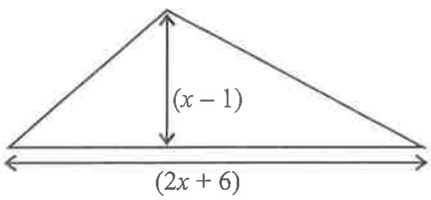


Diagram not accurately drawn

[Total 3 marks]

5 Expand and simplify:

a) $(4m - 3n)(3m + n)$

....

[3]

b) $(n - 3)^3$

....

[3]

[Total 6 marks]

6 Expand and simplify $(2x + 3)(x^2 - 2x + 2)$.

....

[Total 3 marks]

7 Expand and simplify $(3x + 2)(x + 1)(x + 2) - 3x(x^2 - 2x + 4)$.

....

[Total 5 marks]

Exam Practice Tip

If you're struggling with double brackets, don't forget you can always use the FOIL method — multiply the First term in each bracket together, then multiply the Outside terms together, then the Inside terms, and finally multiply together the Last term in each bracket... easy.

Score

32

Factorising

1 Factorise fully $8a^2 - 48ab$.

....

[Total 1 mark]

2 Factorise the following expressions fully.

a) $16x + 4x^2$

....

[1]

b) $25y^2 - 40y^3$

....

[2]

c) $6v^2w^3 + 30v^4w^2$

....

[2]

[Total 5 marks]

3 Factorise the following expressions fully.

a) $4x^2 - 25$

....

[2]

b) $72n^4 - 200m^2$

....

[3]

[Total 5 marks]

Use the difference of two squares to answer question 4.

4 **Factorise the following expressions fully:**

a) $16a^4b^2 - 49c^6$

....

[2]

b) $8x^3 - 18xy^2$

....

[3]

[Total 5 marks]

5 **Factorise fully** $(2x + 3y)^3 + (2x + 3y)^2(x - y)$.

....

[Total 2 marks]

6 **Factorise fully** $(3x + y)^2 - (x + y)^2$.

....

[Total 3 marks]

7 **Factorise fully** $(4x^2 - 1) - (2x - 1)(x + 1)$.

....

[Total 4 marks]

Score: ☐ / **25**

☹ ☐ 😐 ☐ 😉 ☐

Manipulating Surds

1 Write $(6 + \sqrt{2})(1 - \sqrt{2})$ in the form $a + b\sqrt{2}$, where a and b are integers.

....

[Total 2 marks]

2 Express $\frac{5+\sqrt{5}}{3-\sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are integers.

....

[Total 5 marks]

3 Write $2\sqrt{175} - \sqrt{28}$ in the form $a\sqrt{b}$, where a and b are integers.

....

[Total 2 marks]

4 Work out the value of a if $\frac{\sqrt{3}}{2+\sqrt{3}} = \frac{a}{\sqrt{3}}$.

Give your answer in the form $p + q\sqrt{3}$, where p and q are integers.

....

[Total 4 marks]

Score: ____ / 13

Solving Equations

1 Solve the following equations.

a) $7b - 5 = 3(b + 1)$

$b =$ ....

[3]

b) $\frac{45 - z}{5} = 6$

$z =$ ....

[2]

[Total 5 marks]

2 Solve this equation.

$\frac{5 - 3x}{2} + \frac{6x + 1}{5} = 15$

$x =$ ....

[Total 3 marks]

3 The quadrilateral below has a perimeter of 67 cm.

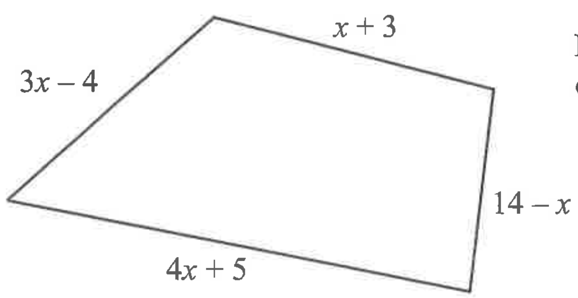


Diagram NOT drawn to scale

All of the lengths on this diagram are in cm. Find the value of x.

$x =$ ....

[Total 3 marks]

Exam Practice Tip

It's a good idea to check your solutions by substituting them back into the equation and checking that everything works out properly. It certainly beats sitting and twiddling your thumbs or counting sheep for the last few minutes of your exam.

Score

11

Rearranging Formulas

1 The relationship between x and y is given by the formula $y = \frac{x-6}{4}$.

a) Rearrange this formula to make x the subject.

....

[2]

b) Find the value of x when $y = 11$.

$x =$ ....

[2]

[Total 4 marks]

2 Rearrange the formula $s = \frac{1}{4}g^2t^2$ to make t the subject.

....

[Total 3 marks]

3 The relationship between a, b and y is given by the formula $a - y = \frac{2b+y}{3a}$.

a) Rearrange this formula to make y the subject.

....

[4]

b) Find the value of y when $a = 3$ and $b = 4$.

$y =$ ....

[2]

[Total 6 marks]

4 Rearrange the formula below to make n the subject.

$$x = \sqrt{\frac{(2+n)}{(1-3n)}}$$

....

[Total 5 marks]

5 Make h the subject of the formula $\frac{W}{Q} = 1 - \frac{c}{h}$.

....

[Total 4 marks]

6 Make a the subject of the formula $2ay - 5 = 7a + 2y$.

....

[Total 3 marks]

7 Make r the subject of the formula $2y = 5 - \frac{\sqrt[3]{r}}{t}$.

....

[Total 3 marks]

8 There are x counters in a bag. The number of counters is increased by half, then y counters are removed from the bag. The number of counters left in the bag is 15.

Find an expression for x in terms of y.

....

[Total 3 marks]

Score: ☐ / **31**

☐ ☐ ☐

Factorising Quadratics

1 Factorise the expression $x^2 + 12x + 35$.

....

[Total 2 marks]

2 Factorise the expressions below.

a) $y^2 + 2y - 24$

....

[2]

b) $3x^2 - 13x - 10$

....

[2]

[Total 4 marks]

3 The equation $x^2 - 11x + 30 = 0$ is an example of a quadratic equation.

a) Factorise the expression $x^2 - 11x + 30$.

....

[2]

b) Use your answer to part a) to solve the equation $x^2 - 11x + 30 = 0$.

$x =$ .... or $x =$ ....

[1]

[Total 3 marks]

4 Solve the equation $x^2 + 6x - 27 = 0$.

$x =$ .... or $x =$ ....

[Total 3 marks]

5 Factorise $4n^2 + 16n + 15$.

....

[Total 2 marks]

6 Fully factorise the expression $6t^2 - 7tu - 20u^2$.

....

[Total 3 marks]

7 The equation $2x^2 + x - 45 = 0$ is an example of a quadratic equation.

a) Fully factorise the expression $2x^2 + x - 45$.

....

[2]

b) Use your answer to part a) to solve the equation $2x^2 + x - 45 = (2x - 9)^2$.

$x =$ .... or $x =$ ....

[4]

[Total 6 marks]

8 The expression $5x^2 - 16x + 12$ is an example of a quadratic expression.

a) Fully factorise the expression $5x^2 - 16x + 12$.

....

[2]

b) Use your answer to part a) to factorise the expression $5(x + 1)^2 - 16(x + 1) + 12$.

....

[2]

[Total 4 marks]

Exam Practice Tip

In the exam, you can check that you've factorised an expression properly by expanding the brackets back out. You should get the same expression that you started with. If you don't then something must have gone wrong somewhere down the line and you'll need to give it another go. Sorry about that.

Score

27

The Quadratic Formula

1 Solve the quadratic equation $x^2 + 7x + 2 = 0$, giving your answers to 2 decimal places.

The quadratic formula will be on your formula sheet.

$x =$ .... or $x =$ ....

[Total 3 marks]

2 Solve the equation $x^2 + 5x - 8 = 0$. Give your answers correct to 2 decimal places.

$x =$ .... or $x =$ ....

[Total 3 marks]

3 Lily is solving an equation of the form $ax^2 + bx + c = 0$. She correctly substitutes the values of a, b and c into the quadratic formula. Her working is shown below.

$$x = \frac{9 \pm \sqrt{(81 - 48)}}{6}$$

Find the values of a, b and c.

$a =$ ...., $b =$ ...., $c =$ ....

[Total 3 marks]

4 Solve the equation $5x^2 - 6x - 3 = 0$. Give your answers in simplified surd form.

$x =$ .... or $x =$ ....

[Total 4 marks]

Exam Practice Tip

One thing you really need to watch out for when it comes to using the quadratic formula are those pesky minus signs. Make sure you don't miss them — it'll mess up your whole answer if you do. And watch out for that sneaky minus sign at the start of the quadratic formula — if you forget that one, you're in trouble.

Score

13

Completing the Square

1 Write the expression $x^2 - 8x + 6$ in the form $(x + a)^2 + b$.

....

[Total 3 marks]

2 Given that $x^2 + ax + b = (x + 3)^2 - 5$, work out the values of a and b.

$a =$ .... and $b =$ ....

[Total 2 marks]

3 A curve has equation $y = 2x^2 - 4x + 11$.

a) Write the expression $2x^2 - 4x + 11$ in the form $a(x + b)^2 + c$.

....

[4]

b) Find the coordinates of the minimum point of the graph.

....

[1]

c) State if and where the graph of the quadratic crosses the x-axis.

Think about the minimum value of the graph.

....

[1]

[Total 6 marks]

4 Given that $5x^2 + nx + 14 \equiv m(x+2)^2 + p$, find the values of the integers m, n and p.

$m =$ .... , $n =$ .... and $p =$ ....

[Total 3 marks]

Score: $\frac{\square}{14}$

Algebraic Fractions

1 Simplify the algebraic fraction below as much as possible.

$$\frac{x^2 - 36}{x^2 + 13x + 42}$$

....

[Total 3 marks]

2 Simplify the algebraic fraction below as much as possible.

$$\frac{5x^2 + 3x - 14}{25x^2 - 49}$$

....

[Total 3 marks]

3 Simplify fully $\dfrac{x^2 + 5x + 6}{x^2 - 9} \div \dfrac{4}{3x^2 - 9x}$

....

[Total 5 marks]

4 Simplify fully $\dfrac{3x^2 - 4x - 15}{x - 4} \times \dfrac{3x^3 - 12x^2}{9x^2 - 25}$

....

[Total 4 marks]

5 Solve $\dfrac{28}{2x + 5} + \dfrac{3}{x} = 4$

....

[Total 7 marks]

Score: ___ / **22**

Factorising Cubics

1 $y = x^3 - x^2 - 4x + 4$ is a cubic equation.

a) Factorise $y = x^3 - x^2 - 4x + 4$ completely.

....

[6]

b) Hence solve the equation $x^3 - x^2 - 4x + 4 = 0$.

....

[1]

[Total 7 marks]

2 $f(x) = x^3 - 4x^2 - 3x + 18$ is a cubic equation.

a) Use the Factor Theorem to show that $(x + 2)$ and $(x - 3)$ are factors of $f(x)$.

....

[2]

b) Use your answer to part a) to simplify $\dfrac{x^3 - 6x^2 + 9x}{x^3 - 4x^2 - 3x + 18}$.

....

[3]

[Total 5 marks]

3 $(x - 1)$ and $(x + 4)$ are factors of the cubic $x^3 + ax^2 + 11x + b$.

Work out the values of a and b.

....

[Total 5 marks]

4 $f(x) = x^3 - 7x - 6$ is a cubic equation.

a) Use the Factor Theorem to show that $(x - 3)$ is a factor of $f(x)$.

....

[1]

b) Find all the solutions of $f(x) = 0$.

....

[4]

[Total 5 marks]

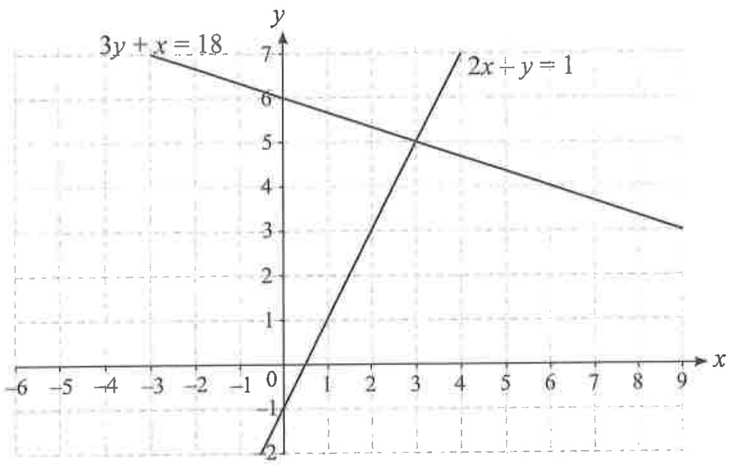
Score: $\dfrac{\quad}{22}$

Simultaneous Equations and Graphs

1 The diagram below shows graphs of $2x - y = 1$ and $3y + x = 18$.

Use the diagram to solve these simultaneous equations:

$2x - y = 1$
$3y + x = 18$



$x =$ .... $y =$ ....

[Total 1 mark]

2 The diagram shows graphs of $y = 18 - 3x$ and $y = 2x - 2$.

a) Find the coordinates of point *P*.

....

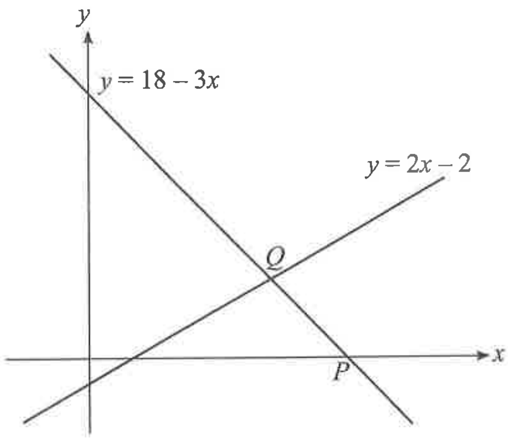
[1]

b) Find the coordinates of point *Q*.

....

[3]

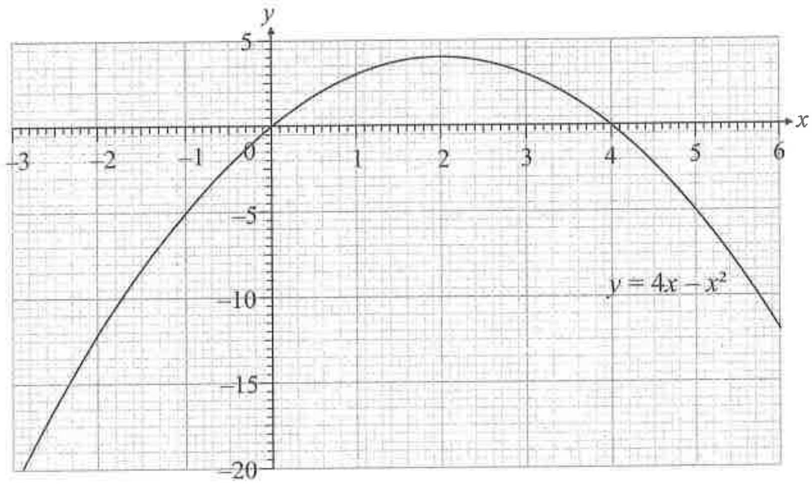
[Total 4 marks]



3 The diagram to the right shows part of the graph of $y = 4x - x^2$.

Use the graph to solve these simultaneous equations.

$y = 2x - 3$
$y = 4x - x^2$



$x =$ .... , $y =$ ....

and $x =$ .... , $y =$ ....

[Total 3 marks]

Score: ☐ / 8

☐ ☐ ☐

Simultaneous Equations

1 Solve this pair of simultaneous equations.

$3x + 5y = 49$
$5x + 2y = 31$

$x =$ .... $y =$ ....

[Total 4 marks]

2 Solve the following pair of simultaneous equations.

$\frac{3y-4}{x-2} = 5$ and $3x = 2y + 5$

$x =$ .... $y =$ ....

[Total 5 marks]

3 Solve the simultaneous equations below.

$x^2 + y = 2$
$2x + y = -6$

$x =$ .... , $y =$ ....

and $x =$ .... , $y =$ ....

[Total 5 marks]

4 Solve the following pair of simultaneous equations.

$3x + 2y^2 = 11$
$y = x + 3$

$x =$ .... , $y =$ ....

and $x =$ .... , $y =$ ....

[Total 5 marks]

5 Solve this pair of simultaneous equations.

$3x^2 - y^2 = -22$
$y = x + 4$

$x =$ .... , $y =$ ....

and $x =$ .... , $y =$ ....

[Total 5 marks]

Exam Practice Tip

When you're solving simultaneous equations in the exam, it's always a good idea to check your answers at the end. Just substitute your values for x and y back into the original equations and see if they add up as they should. If they don't then you must have gone wrong somewhere, so go back and check your working.

Score

24

Inequalities

1 Find the integer values of p which satisfy the inequality $12 < 5p \leq 30$.

....

[Total 3 marks]

2 Solve the following inequalities.

a) $6q - 8 < 40$

....

[2]

b) $\frac{3x}{4} \leq 9$

....

[2]

[Total 4 marks]

3 Solve the following inequalities.

a) $7x - 2 < 2x - 42$

....

[2]

b) $9 - 4x > 17 - 2x$

....

[2]

[Total 4 marks]

4 Solve the following inequalities.

a) $3 \leq 2p + 5 \leq 15$

....

[2]

b) $q^2 - 9 > 0$

....

[3]

[Total 5 marks]

5 Solve $w^2 \geq 25$.

....

[Total 2 marks]

6 Solve the following inequalities.

a) $3x^2 - 5x - 2 \leq 0$

....

[3]

b) $2x^2 > x + 1$

....

[3]

[Total 6 marks]

7 Given that $-8 \leq x \leq 2$, complete the inequality below for $(x + 1)^2$.

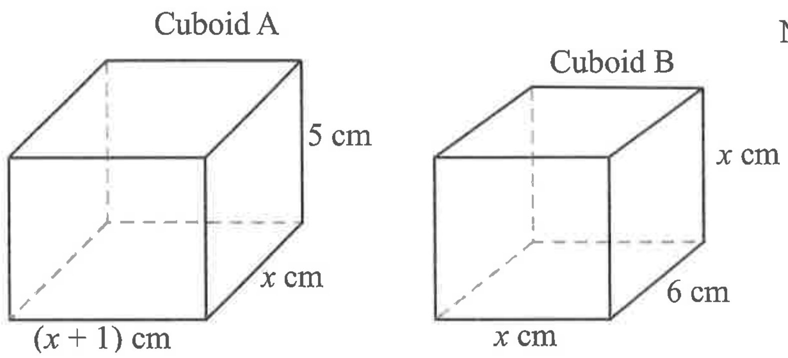
Think about the biggest and smallest possible values of $(x + 1)^2$.

.... $\leq (x + 1)^2 \leq$ ....

[Total 3 marks]

8 Look at the two cuboids below. The volume of cuboid A is greater than the volume of cuboid B.

Not drawn accurately



a) Show that $x^2 - 5x < 0$.

[3]

b) If x is an integer, find the greatest possible volume of cuboid A.

.... cm^3

[4]

[Total 7 marks]

Score: ☐ / **34**

Algebraic Proof

1 Prove that $(2n + 1)^2 - 5(2n + 1)$ is always an even number for integer values of n.

[Total 2 marks]

2 A linear sequence begins 3, 9, 15, 21... The nth term of this sequence is $6n - 3$.

Prove algebraically that the sum of two consecutive terms in this sequence is always a multiple of 12.

[Total 2 marks]

3 Prove that the square of any odd number is always one more than a multiple of 4.

[Total 3 marks]

4 If n is a positive integer, prove that $(2n + 5)^2 - (n + 1)^2$ is always a multiple of 3.

[Total 3 marks]

5 Prove that the product of three consecutive even numbers is always a multiple of 8.

[Total 3 marks]

Score: ___ / **13**

Sequences

1 Find an expression for the nth term of the quadratic sequence 4, 6, 10, 16, 24...

....

[Total 5 marks]

2 Find an expression for the nth term of the sequence 13, 18, 25, 34, 45...

....

[Total 5 marks]

3 A sequence begins $\frac{3}{4}, \frac{5}{7}, \frac{7}{10}, \frac{9}{13}, ...$

a) Find an expression for the nth term of the sequence.

Treat the numerators and denominators as separate sequences.

....

[3]

b) What is the limiting value of the sequence as $n \to \infty$?

....

[2]

[Total 5 marks]

4 The nth term of a sequence is $\frac{an + 1}{bn + 3}$.

The first term of the sequence is 1, and the limiting value as $n \to \infty$ is 2.
Work out the 6th term in the sequence.

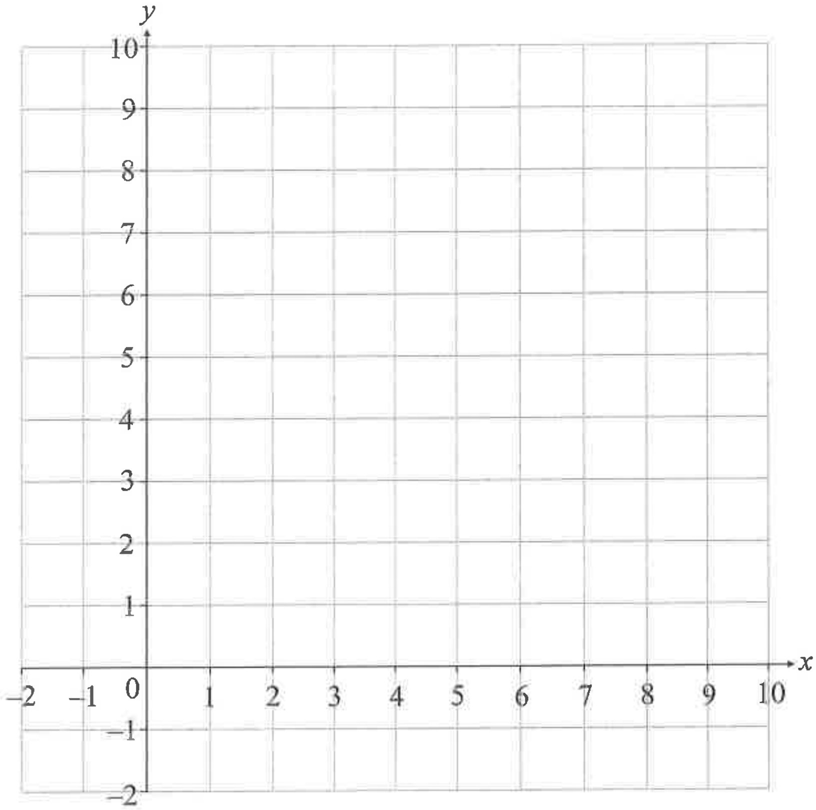
....

[Total 5 marks]

Score: $\frac{\quad}{20}$

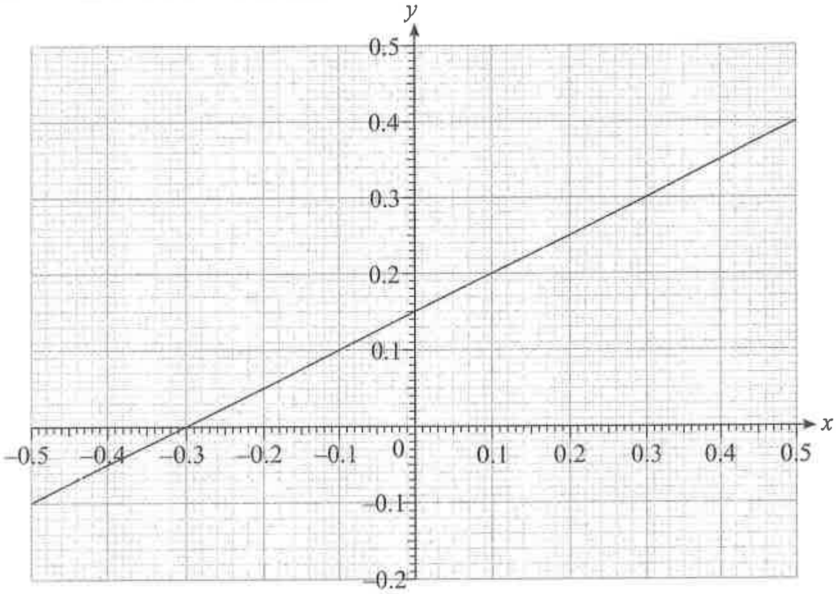
Straight-Line Graphs

1 Draw the graph $3y + x = 9$ on the axes below, for values of x in the range $-2 \le x \le 10$.



[Total 2 marks]

2 The diagram below shows a straight line.

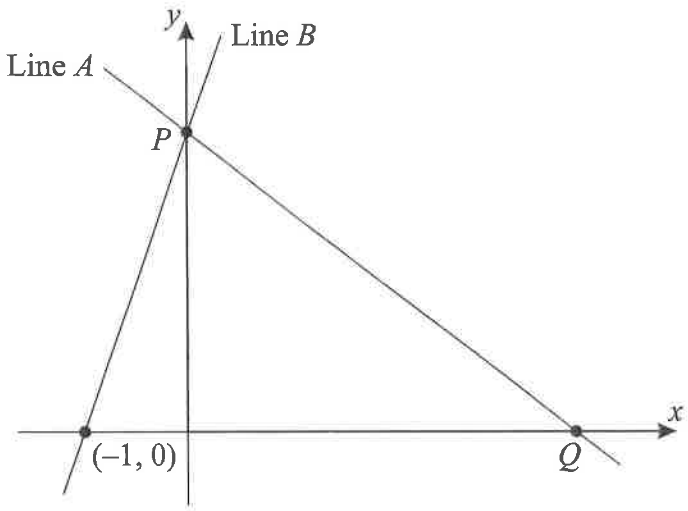


Find the equation of the line.

....

[Total 2 marks]

3 The graphs of two equations are sketched below. The equation of line A is $3x + 4y = 12$.



a) Find the gradient of line A.

....

[1]

b) Find the coordinates of point Q.

Q (.................... , ...)

[1]

c) Given that both lines pass through point P on the y-axis, find the equation of line B.

....

[2]

[Total 4 marks]

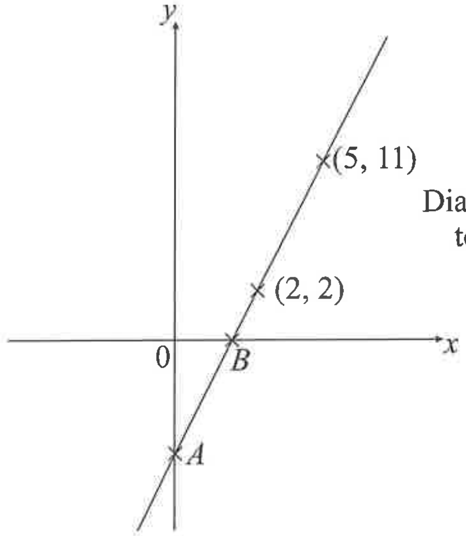
4 **Find the equation of the line which passes through the points (–1, 17) and (5, –7).**

Give your answer in the form $y = mx + c$.

....

[Total 2 marks]

5 The graph below shows a straight line which passes through the points (2, 2) and (5, 11).



a) Find the coordinates of A and B.

A (.................... , ...)

B (.................... , ...)

[4]

b) Write down the equation of the line which is parallel to the line above and passes through the point (4, –8).

....

[2]

[Total 6 marks]

6 The lines $y = 5x + 3$ and $y = 3x + 7$ intersect at the point M.

Line N goes through point M and is perpendicular to the line $y = 4x - 5$.
Find the equation of line N.

....

[Total 5 marks]

Score:

21

Coordinates and Ratio

1 Point A has coordinates $(1, k)$ and point B has coordinates $(6, 3)$.
The line through A and B has equation $5y - x = 9$.

a) Find the value of k.

....
[1]

b) Find the mid-point of AB.

....
[2]

[Total 3 marks]

2 A line segment joins the points A $(-5, -9)$ and B $(9, 12)$.
Point C lies on this line segment such that $AC : CB = 3 : 4$.

Work out the coordinates of C.

....
[Total 4 marks]

3 P and Q are points with coordinates (a, a) and $(b, -2)$ respectively.
R $(-3, 1)$ is a point on the line segment PQ such that $QR : RP$ is $1 : 2$.

Work out the values of a and b.

a = ... b = ...
[Total 5 marks]

Score: ☐ / **12**

☐ ☐ ☐

Functions

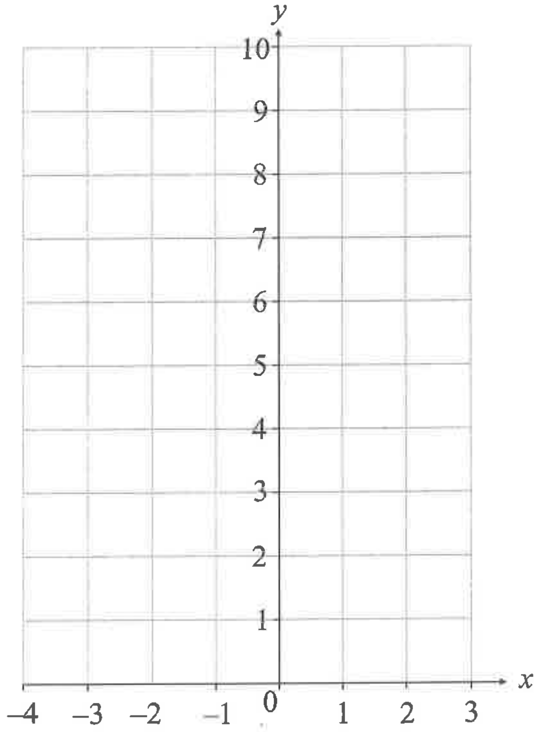
1 A function f(x) is defined by the expressions below.

$$f(x) = -2x \quad \text{for } x < -1$$
$$= 2 \quad \text{for } -1 \le x \le 1$$
$$= x^2 + 1 \quad \text{for } x > 1$$

a) Find the value of f(–3.5).

....
[1]

b) Draw the graph of $y = f(x)$ for values of x between –4 and 3.



[3]

c) Solve the equation f(x) = 26.

x = ...
[3]
[Total 7 marks]

2 A function f(x) is given by $f(x) = \sqrt{2x + 7}$.

Determine the values of x which are excluded from the domain of f.

....
[Total 2 marks]

3 A function g(x) is defined by $g(x) = 3 - x^2$.

a) Find g(3).

....

[1]

b) Write down the range of g(x).

....

[1]

[Total 2 marks]

4 f(x) is a decreasing function with domain $-3 \leq x < 6$ and range $-27 < f(x) \leq 9$.

Given that $f(x) = -nx - m$, find m and n.

m = ... n = ...

[Total 4 marks]

5 A function g(x) is defined as $g(x) = 8 - 3x$, for $-2 \leq x < 4$.

a) Work out the range of g(x).

....

[2]

b) Find and simplify an expression for $g(2 - x)$.

....

[2]

c) Solve $g(2 - x) = -1$

....

[1]

[Total 5 marks]

Exam Practice Tip

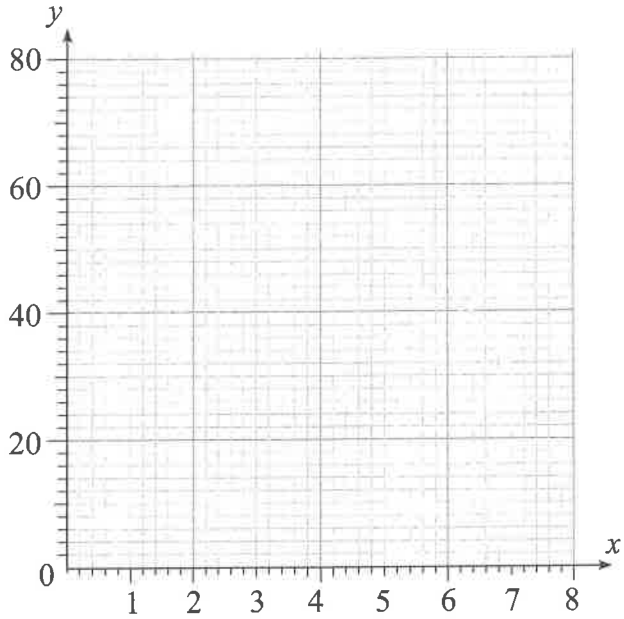
Function notation and the vocabulary of 'domain' and 'range' is something which people often struggle with in exams. That doesn't stop the examiners from asking questions on functions all the bloomin' time. Remember, the domain is the set of numbers the function maps from, and the range is the set of numbers it maps to. Easy, right?

Score

20

Quadratic Graphs

1 Two variables are related by the equation $y = 32x - 4x^2$.



a) Draw the graph of $y = 32x - 4x^2$ on the grid.

[2]

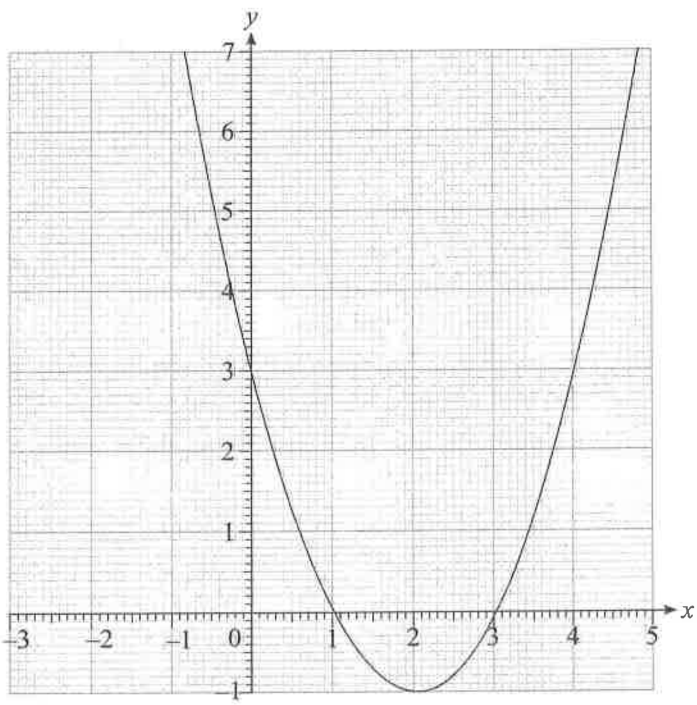
b) Using the graph, give the maximum value of y.

....

[1]

[Total 3 marks]

2 The graph below shows $y = x^2 - 4x + 3$.



a) Find the values of x when $x^2 - 4x + 3 = 0$.

x = ... and x = ...

[1]

b) Find the solutions of $x^2 - 5x + 4 = 0$ by drawing an appropriate linear graph.

x = ... and x = ...

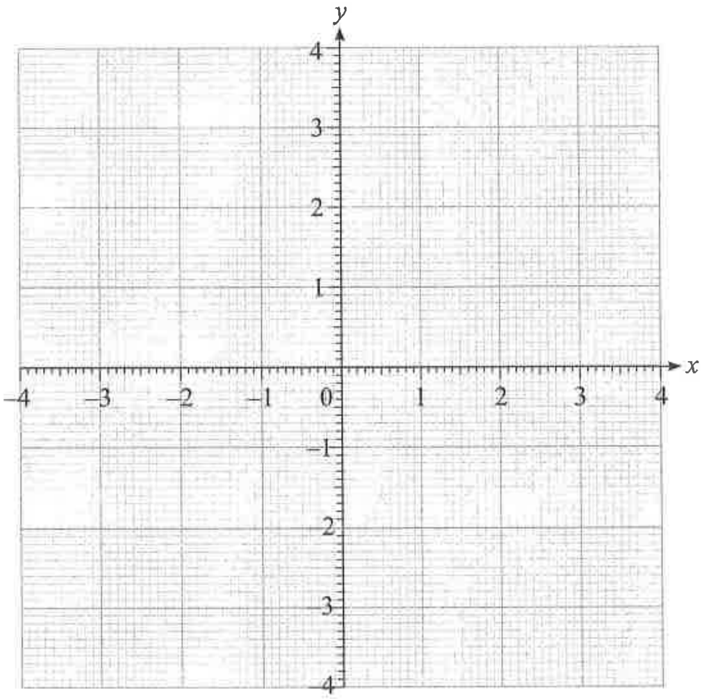
[4]

[Total 5 marks]

Score: ____ / 8

Equation of a Circle

1 Draw the graph of $x^2 + y^2 = 9$ on the grid below.



[Total 2 marks]

2 C is a circle with the equation: $x^2 + y^2 - 2x - 10y + 21 = 0$.

Find the centre and radius of C. Where appropriate, give your answers in surd form.

Centre = (.............., ...)

Radius = ...

[Total 5 marks]

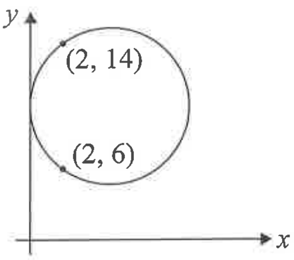
3 A circle has centre (2, 7).
The point (2, 10) lies on the circumference of the circle.

Write down the equation of the circle.

....

[Total 3 marks]

4 A circle has radius 5 and passes through the points (2, 6) and (2, 14). The circle also touches the y-axis, as shown.



a) Write down the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$.

....

[4]

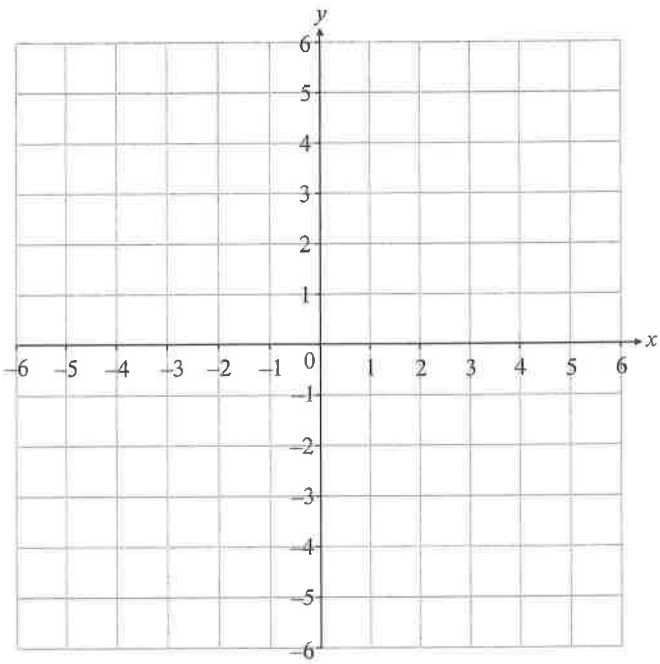
b) Show algebraically that the point (8, 14) lies on the circle.

[1]

[Total 5 marks]

5 A circle has equation $(x - 2)^2 + y^2 = 9$ and a line has equation $y = 5 - 2x$.

a) Draw the graphs of the circle and the line on the grid.



[3]

b) Shade the region where $(x - 2)^2 + y^2 \leq 9$ and $y \leq 5 - 2x$.

[1]

[Total 4 marks]

Score: ☐ / 19

Differentiation

1 A curve has equation $y = 2x^3 + x^2 - 8x + 3$.

Find $\frac{dy}{dx}$.

$\frac{dy}{dx} =$ ....

[Total 1 mark]

2 Find the gradient of the graph of $y = -x^2 + 3$ at $x = -1$ and $x = 2$.

....

[Total 3 marks]

3 A curve is given by the equation $y = 4x^3 - 2x + 3$.

a) Find the gradient function for the curve.

....

[1]

b) Find the x-coordinates of the points on the curve where the gradient is equal to 1.

$x =$ .... and $x =$ ....

[3]

[Total 4 marks]

4 A curve has equation $y = \frac{5x^3(x^2+2)}{(\sqrt{x})^2}$.

Find $\frac{dy}{dx}$.

$\frac{dy}{dx} =$ ....

[Total 5 marks]

5 Find the rate of change of y with respect to x at the point $(-1, -1)$ for the curve $y = x(2x + 1)^2$.

....

[Total 6 marks]

Score: ____ / **19**

Finding Tangents and Normals

1 The curve C is given by the equation $y = 2x^3 - 4x^2 - 4x + 12$.

a) Find $\frac{dy}{dx}$.

$\frac{dy}{dx} =$ ....

[2]

b) Write down the gradient of the tangent to the curve at the point where $x = 2$.

....

[1]

c) Hence or otherwise find an equation for the normal to the curve at this point.

....

[3]

[Total 6 marks]

2 A curve has the equation $y = x(x - 2)(x + 2)$.

Find the equation for the tangent to the curve at the point where $x = 1$.

....

[Total 5 marks]

3 The diagram shows the graph of $y = 4 + 4x - x^2$.

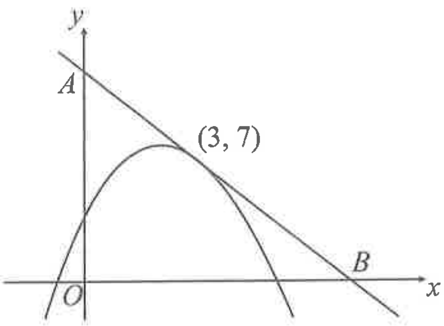


Diagram not accurately drawn

The tangent to the curve at the point (3, 7) cuts the axes at A and B.
Work out the area of triangle OAB.

....

[Total 6 marks]

Score: $\frac{}{17}$

Stationary Points

1 $y = (x + 4)(x - 8)$ is the equation of a curve.

Find the coordinates of the turning point of this curve.

(.............., ...)

[Total 3 marks]

2 A curve has equation $y = 6 + \frac{2x^3 - 6x^2 + 6x}{3}$.

a) Find $\frac{dy}{dx}$ for the curve.

$\frac{dy}{dx} =$ ....

[2]

b) Hence find the coordinates of the stationary point on the curve.

(.............., ...)

[3]

c) Determine the nature of the stationary point.

....

[2]

[Total 7 marks]

3 A curve is given by the equation $y = x(x - 1)^2$.

a) Determine the coordinates of the stationary points for the curve.

....

[6]

b) Find whether each of these points is a maximum or minimum.

....

[3]

c) Show that the function is increasing when $x = 2$.

[1]

[Total 10 marks]

4 A curve has equation $y = x^3 - 6x^2 + 12x + 5$.

a) Show that the curve has exactly one stationary point.

[4]

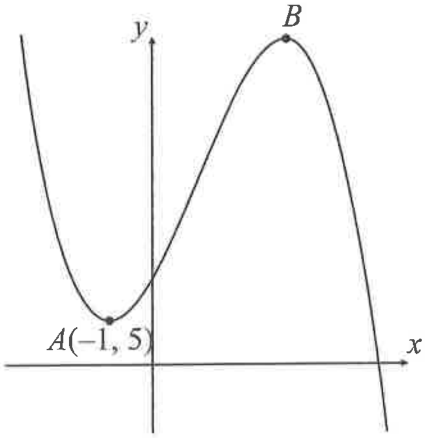
b) By considering the gradient of the curve either side of the stationary point, determine the nature of the stationary point.

....

[2]

[Total 6 marks]

5 The graph shows the graph of $y = f(x)$ for the function $f(x) = -x^3 + 3x^2 + ax + b$.



The curve has a stationary point at $A(-1, 5)$.

a) Work out the values of a and b.

$a =$ ...., $b =$ ....

[5]

b) Find the x-coordinate of the other stationary point, B.

$x =$ ....

[2]

c) Write down the range of x-values for which the function is decreasing.

....

[2]

[Total 9 marks]

Score: $\frac{\square}{35}$

Curve Sketching

1 $f(x) = x^3 - 3x^2 - 1$.

a) Find the coordinates of any stationary points on the curve $y = f(x)$.

....

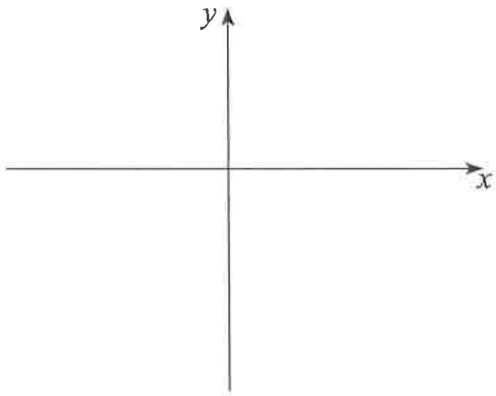
[5]

b) Determine the nature of these stationary points.

....

[3]

c) Sketch the curve on these axes.



[3]

[Total 11 marks]

2 $f(x) = (x + 2)(a - x)$, where a is an integer. The curve $y = f(x)$ has a stationary point when $x = 2$.

a) Work out the value of a.

a = ...

[3]

b) By considering the nature of the stationary point, explain why the equation $f(x) = 20$ has no real solutions.

....

....

[2]

c) Sketch the curve on a set of axes. Clearly label the points of intersection with the axes and the stationary point.

[3]

[Total 8 marks]

Score: ____ / **19**

Matrices

1 Find the result of each matrix calculation below.

a) $3\begin{pmatrix} 2 & 5 \\ 1 & 7 \end{pmatrix}$

....
[1]

b) $-5\begin{pmatrix} -2 & 5 \\ 11 & 3 \end{pmatrix}$

....
[1]

c) $\begin{pmatrix} 4 & 3 \\ 7 & 2 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ 3 & 9 \end{pmatrix}$

....
[2]

d) $\begin{pmatrix} 7 & -1 \\ 12 & 2 \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 2 & 5 \end{pmatrix}$

....
[2]

[Total 6 marks]

2 $\mathbf{M} = \begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix}$ $\mathbf{N} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

Work out **MN**.

....
[Total 2 marks]

3 $\begin{pmatrix} 0 & x \\ -\frac{1}{2} & 1 \end{pmatrix}\begin{pmatrix} \frac{2}{3} & -2 \\ y & 0 \end{pmatrix} = \mathbf{I}$

Find x and y.

x = ..., y = ...
[Total 4 marks]

Exam Practice Tip

Multiplying two matrices really isn't that bad once you've got the hang of it — you just multiply the rows of the first matrix by the columns of the second one and add all the numbers up. The tricky bit is getting the entries of your answer matrix in the right place. The only way to really get up to speed with this is to do tons of practice.

Score
12

Matrix Transformations

1 The matrix $\begin{pmatrix} 3 & -2 \\ 1 & 2 \end{pmatrix}$ maps the point $(a, 5)$ to the point $(2, b)$.

This mapping can be represented by the matrix equation $\begin{pmatrix} 3 & -2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} a \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ b \end{pmatrix}$

Work out the values of a and b.

a = ..., b = ...

[Total 3 marks]

2 The matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ represents a transformation.

Describe the transformation fully.

....

[Total 2 marks]

3 The matrix $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ represents a rotation of 180° about the origin.

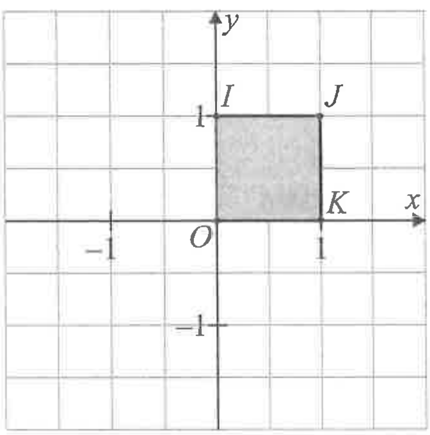
The matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ represents a reflection in the line $y = -x$.

Work out the matrix that corresponds to a rotation of 180° about the origin followed by a reflection in the line $y = -x$.

....

[Total 2 marks]

4 *OIJK* is the unit square.



The unit square is first transformed by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ then by the matrix $\begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}$.

Draw the image of the unit square after both transformations.

[Total 4 marks]

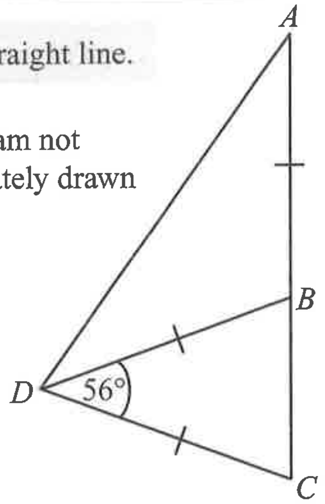
Score: $\frac{\square}{11}$

Geometry

1 *ABD* and ACD are isosceles triangles with *AB* = *BD* = *CD*. *ABC* is a straight line.

Work out the size of angle *BAD*.

Diagram not accurately drawn



..................................°

[Total 3 marks]

2 *ACDH* is a quadrilateral. *BCDE*, *GHD* and *AHF* are straight lines.

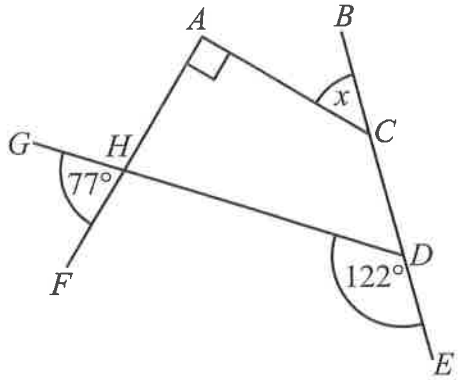


Diagram not accurately drawn

Work out the size of the angle marked *x*.

..................................°

[Total 4 marks]

3 *ABC* is a scalene triangle.

Work out the size of angle *ACB*.

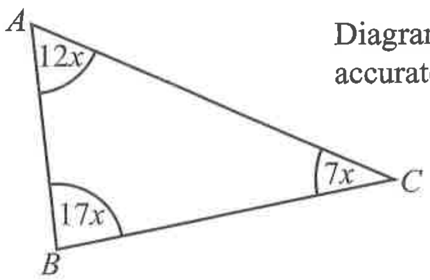


Diagram not accurately drawn

..................................°

[Total 3 marks]

4 *AGF* and *BD* are parallel lines. *AEDC* and *BEG* are straight lines.

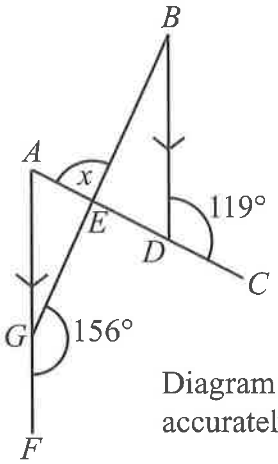


Diagram not accurately drawn

Work out the size of angle x.
Give reasons for each stage of your working.

....°

[Total 4 marks]

5 *AED* and *FEC* are straight lines that cross at *E*. *AB* and *BCD* are perpendicular to each other.

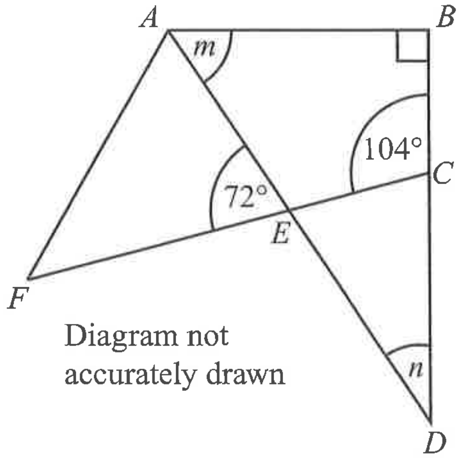


Diagram not accurately drawn

a) Find angle m.
Give a reason for each stage of your working.

$m =$ ....°

[2]

b) Show that $n = 32°$.

[1]

[Total 3 marks]

6 *RSTU* is a quadrilateral. Lines *RS* and *UT* are parallel.

Find the values of x and y.

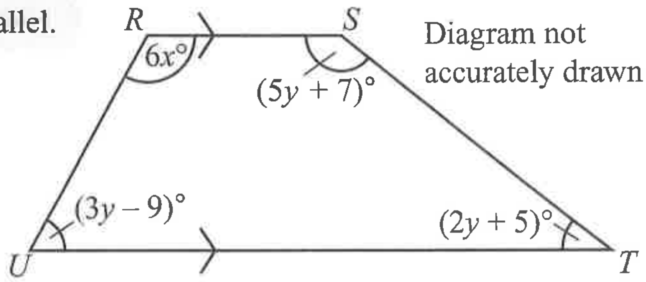


Diagram not accurately drawn

$x =$ ....

$y =$ ....

[Total 5 marks]

Exam Practice Tip

If you find yourself staring at a geometry problem in the exam not knowing where to start, just try finding any angles you can — don't worry tooooo much at first about the particular angle you've been asked to find. Just make sure you make it really clear which angle you're finding at each step. Label the diagram if it helps.

Score

22

Area

1 The diagram below shows a field. The area of the field is 23 700 m².

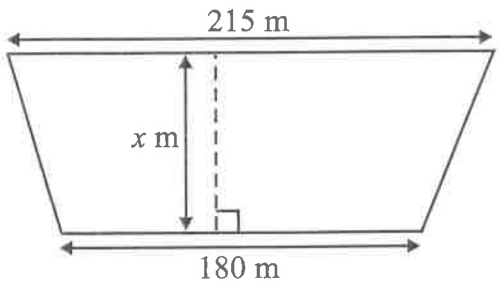


Diagram not accurately drawn

Work out the value of x.

....

[Total 3 marks]

2 The circle below has a radius of 30 cm.
The sector S has a central angle of 80°.

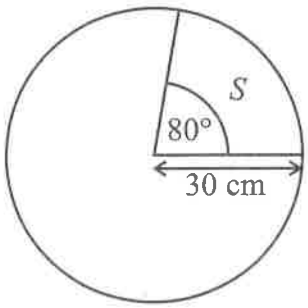


Diagram not accurately drawn

a) Find the exact area of the sector S of the circle.

.... cm²

[2]

b) Find the perimeter of the sector S of the circle.
Give your answer to 3 significant figures.

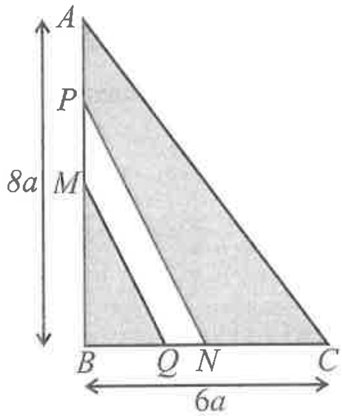
.... cm

[3]

[Total 5 marks]

3 ABC is a right-angled triangle with base $6a$ and height $8a$.

M and N are the midpoints of AB and BC respectively.
$AP : AB = 1 : 4$ and $BQ : BC = 1 : 3$
Find an expression for the shaded area in terms of a.



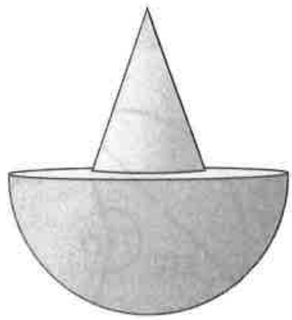
....

[Total 5 marks]

Score: 13

Surface Area and Volume

1 This solid is made from a hemisphere and a cone.

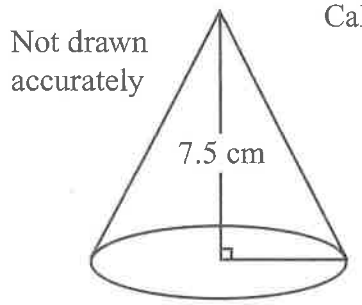


The hemisphere has a diameter of 22 cm.
The slanting length of the cone is 18 cm and the radius of its base is 8 cm.

Work out the total surface area of the solid.
Give your answer in terms of π.

.... cm^2
[Total 6 marks]

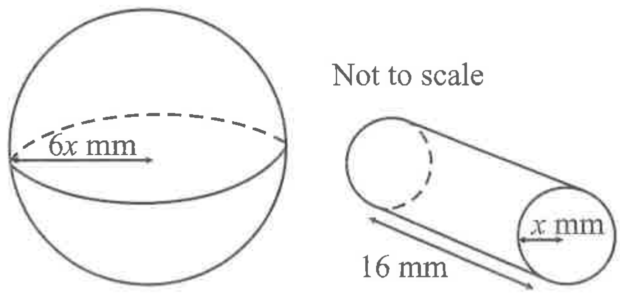
2 The volume of the right circular cone shown below is 40π cm^3 correct to 3 significant figures.



Calculate the radius of the base of the cone.

.... cm
[Total 3 marks]

3 The diagram below shows a solid metal sphere with a radius of $6x$ mm, and a solid metal cylinder with a radius of x mm and length 16 mm.



The sphere is melted down and all of the metal is used to make cylinders with the dimensions shown. 126 cylinders are made. Work out the value of x.

....
[Total 6 marks]

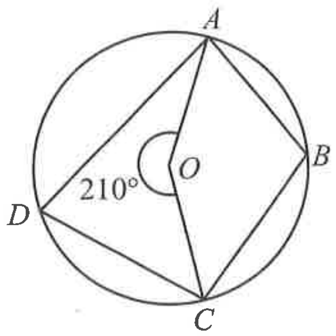
Exam Practice Tip

You'll be given the surface area and volume formulas for a sphere and a cone in your exam paper, but that's it, you're on your own for other types of solid. Make sure you can find the surface area and volume of other solids without looking up the formula — if it's a struggle then just keep practising, it's the only way it'll sink in.

Score
15

Circle Geometry

1 The diagram shows a circle, centre O. A, B, C and D are points on the circumference.



Not to scale

a) Work out the size of angle ADC. Give reasons for your working.

..............................°

[2]

b) Explain why angle $ABC = 105°$.

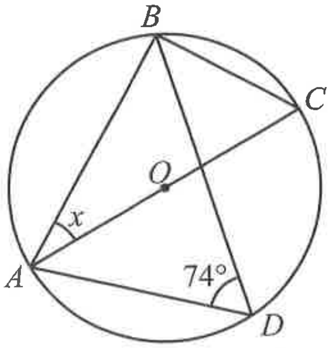
....

....

[1]

[Total 3 marks]

2 The diagram below shows a circle with centre O. A, B, C and D are points on the circumference of the circle and AOC is a straight line.



Not to scale

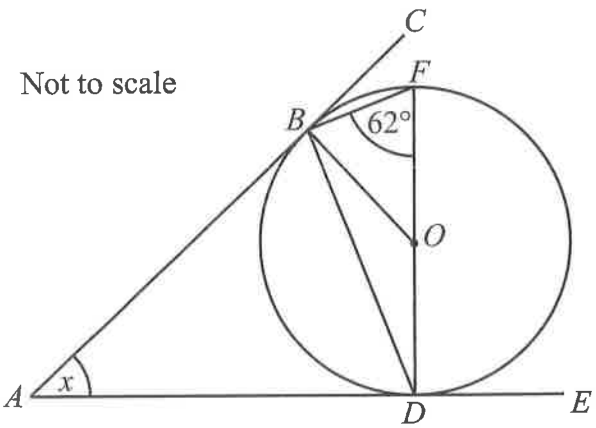
Work out the size of the angle marked x.

$x =$ ....°

[Total 3 marks]

3 The diagram below shows a circle with centre O. ABC and ADE are tangents to the circle and DOF is a straight line. Angle $OFB = 62°$.

Not to scale



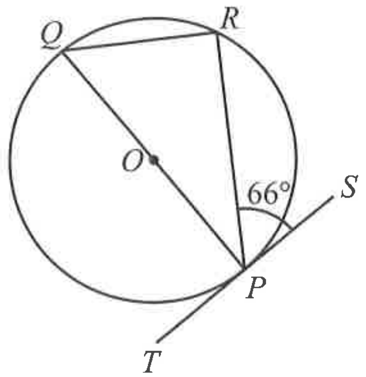
Work out the size of the angle marked x.

Start by finding the size of angle DBF.

$x =$ ....°

[Total 5 marks]

4 The diagram below shows a circle, centre O. P, Q and R are points on the circumference of the circle. SPT is a tangent to the circle. Angle RPS is 66°.



Not to scale

a) Work out the size of angle QPR.

.................................°

[2]

b) Explain why angle PQR = 66°.

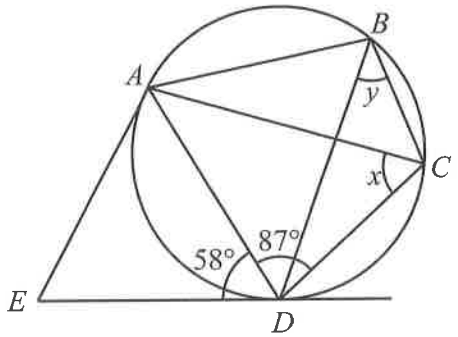
....

....

[1]

[Total 3 marks]

5 A, B, C and D are points on the circumference of a circle, and EA and ED are tangents to the circle. Angle EDA is 58° and angle ADC is 87°.



Not to scale

Work out the size of angles x and y.
Give reasons for each step in your working.

x = ...° y = ...°

[Total 4 marks]

Exam Practice Tip

Make sure you know the rules about circles really, really well. Draw them out and stick them all over your bedroom walls, your fridge, even your dog. Then in the exam, go through the rules one-by-one and use them to fill in as many angles in the diagram as you can. Keep an eye out for isosceles triangles formed by two radii too.

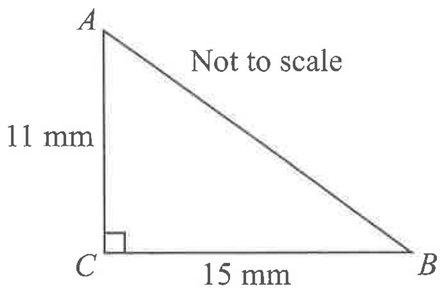
Score

18

Pythagoras' Theorem

1 The diagram shows a right-angled triangle *ABC*. *AC* is 11 mm long. *BC* is 15 mm long.

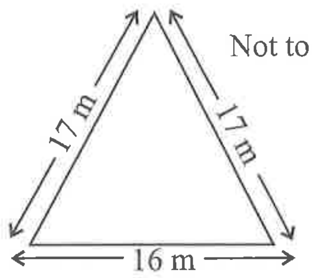
Calculate the length of *AB*.
Give your answer to 3 significant figures.



.... mm
[Total 2 marks]

2 A triangle has a base of 16 m. Its other two sides are both 17 m long.

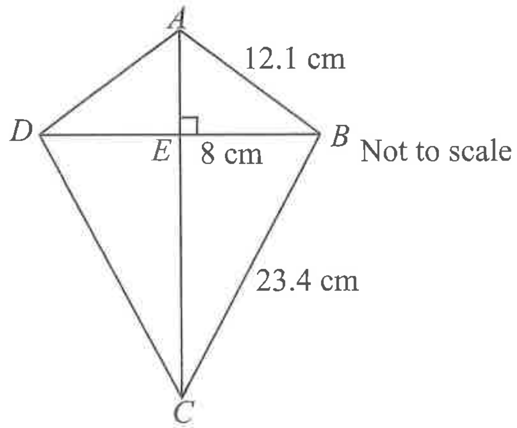
Calculate the area of the triangle.



.... m^2
[Total 2 marks]

3 The diagram shows a kite *ABCD*. *AB* is 12.1 cm long. *BC* is 23.4 cm long. *BE* is 8 cm in length.

Work out the perimeter of triangle *ACD*.
Give your answer to 2 decimal places.



.... cm
[Total 5 marks]

4 Work out the distance between the points (–2, 5) and (5, –4).

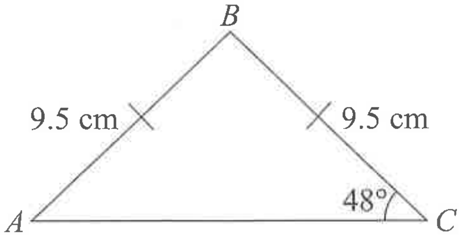
.... units
[Total 2 marks]

Score: / 11

Trigonometry — Sin, Cos, Tan

1 In the triangle ABC, $AB = BC = 9.5$ cm and angle $C = 48°$.

Calculate the length AC.
Give your answer to 2 decimal places.



Not drawn accurately

.... cm

[Total 3 marks]

2 The diagram below is made from two right-angled triangles, ABC and ACD.

$BC = 12$ cm
$\tan y = 0.75$
$\sin x = \frac{2}{3}$

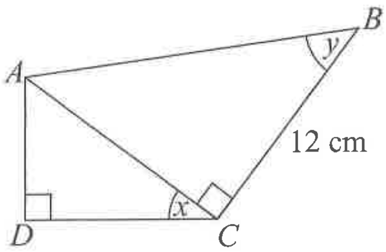


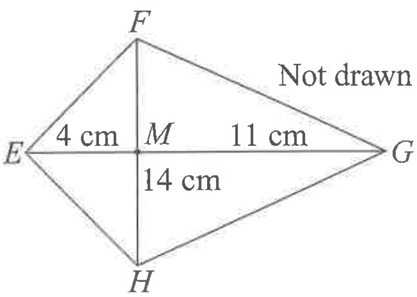
Diagram not drawn accurately

Find the length of AD.

.... cm

[Total 4 marks]

3 The diagram shows a kite $EFGH$.
Diagonal EG bisects the diagonal HF at M.
$EM = 4$ cm, $MG = 11$ cm and $HF = 14$ cm.



Not drawn accurately

a) Calculate the size of angle HGM.
Give your answer to 3 significant figures.

..............................°

[3]

b) Calculate the size of angle FEH.
Give your answer to 3 significant figures.

..............................°

[3]

[Total 6 marks]

Exam Practice Tip

In an exam, it'll help if you start by labelling the sides of a right-angled triangle, opposite (O), adjacent (A) and hypotenuse (H) — these are easy to get muddled up. If you're working out an angle, make sure you check whether it's sensible — if you get an angle of 720° or 0.0072°, it's probably wrong so give it another go.

Score

13

The Sine and Cosine Rules

1 In the triangle below, $BC = 31$ cm, $AC = 37$ cm and angle $ACB = 82°$.

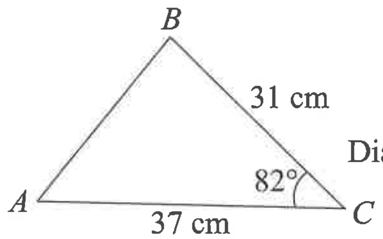


Diagram not accurately drawn

a) Calculate the length of AB.
Give your answer to 3 significant figures.

.... cm
[3]

b) Calculate the area of triangle ABC.
Give your answer to 3 significant figures.

.... cm²
[2]

[Total 5 marks]

2 The diagram below is made up of two triangles. $AB = 10$ m, $BC = 7$ m, angle $ADB = 44°$, angle $BAD = 26°$ and angle $BCD = 62°$.

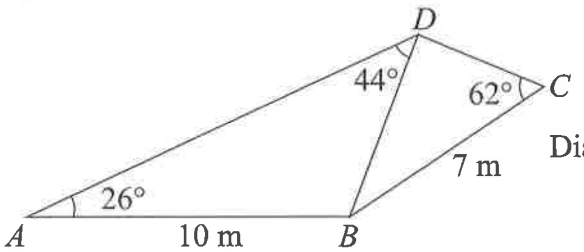


Diagram not accurately drawn

Calculate:

a) the length of BD.
Give your answer to 3 significant figures.

.... m
[3]

b) the size of angle BDC.
Give your answer to 3 significant figures.

..............................°
[3]

[Total 6 marks]

3 ABCD is a trapezium.

BC is parallel to AD.
BC = 12 cm
AD = 46 cm
Angle BAC = 35°

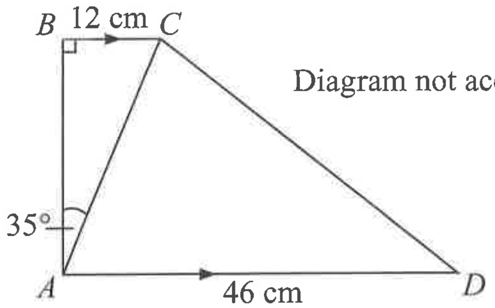


Diagram not accurately drawn

a) Calculate the length of AC.

.... cm
[3]

b) Work out the perimeter of triangle ACD to the nearest cm.

.... cm
[4]

[Total 7 marks]

4 In the triangle shown, AB = 5 cm, BC = 11 cm and AC = 8 cm.

Calculate the area of the triangle.

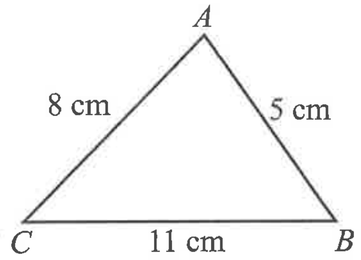


Diagram not accurately drawn

.... cm^2
[Total 5 marks]

5 ABCD is a quadrilateral.

Given that AC = 49 cm, work out the area of ABCD to 3 significant figures.
Show clearly how you get your answer.

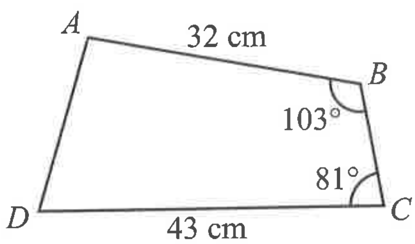


Diagram not accurately drawn

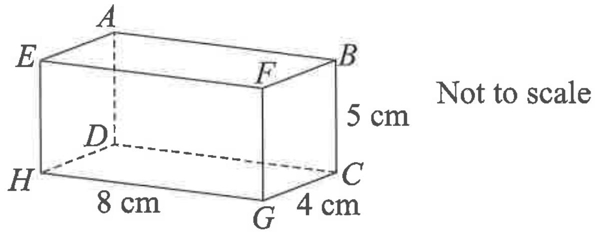
.... cm^2
[Total 8 marks]

Score: ___ / **31**

3D Pythagoras

1 The diagram below shows cuboid *ABCDEFGH*.

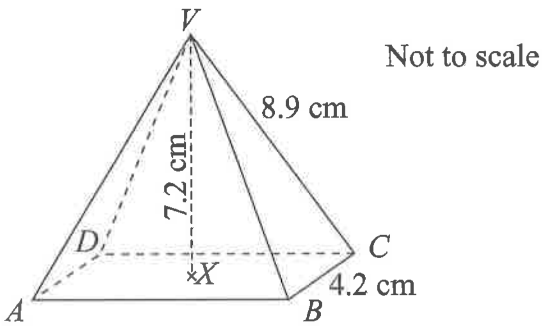
The cuboid has sides of length 8 cm, 5 cm and 4 cm.



Calculate the length of the diagonal *BH*.
Give your answer to 3 significant figures.

.... cm
[Total 2 marks]

2 The diagram shows a pyramid with a rectangular base.
The vertex, *V*, of the pyramid is directly above the centre of the base *ABCD*.



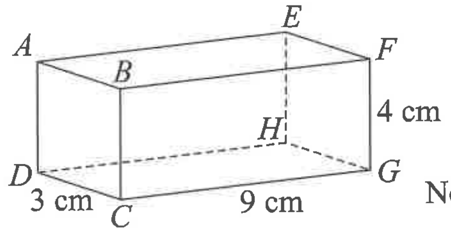
$VC = 8.9$ cm, $VX = 7.2$ cm and $BC = 4.2$ cm.
Work out the length *AB*. Give your answer to 3 significant figures.

.... cm
[Total 4 marks]

Score: ___ / 6

3D Trigonometry

1 The diagram below shows the cuboid *ABCDEFGH* which has sides of length 3 cm, 4 cm and 9 cm.



a) Work out the length of the diagonal *FD*.
Give your answer to 3 significant figures.

.... cm
[2]

b) Find the size of the angle between the line *FD* and the line *DG*.
Give your answer to 1 decimal place.

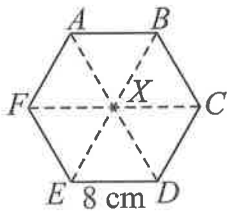
.... °
[2]

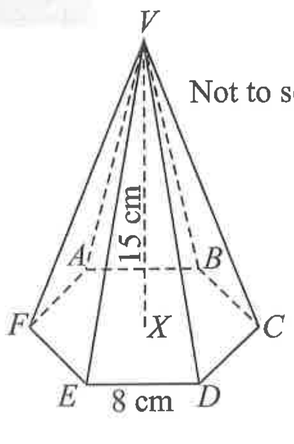
[Total 4 marks]

2 The base of the pyramid *ABCDEFV* is a regular hexagon with side length 8 cm. The vertex, *V*, of the pyramid is directly above the centre of the base.

X is the centre of the base.
The height *VX* of the pyramid is 15 cm.

Calculate the angle between the plane *VED* and the base *ABCDEF*.
Give your answer to 1 decimal place.





.... °
[Total 3 marks]

Score: ☐ / 7

Trig Values

1 Draw a line to match each trigonometric expression on the left with its value on the right.

tan 45°	0.25
sin 30°	0.5
cos 30°	$\frac{\sqrt{3}}{2}$
tan 60°	1
	$\sqrt{3}$

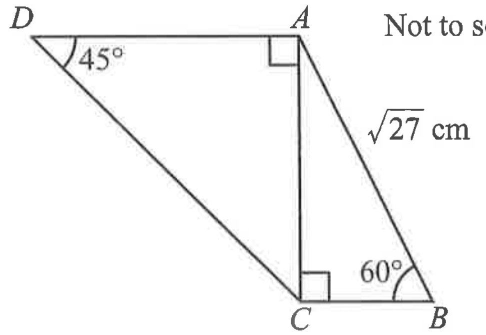
[Total 2 marks]

2 The diagram shows two triangles, *ABC* and *ACD*.

$AB = \sqrt{27}$ cm
Angle $ABC = 60°$
Angle $ADC = 45°$

Prove that $AD = \frac{9}{2}$ cm.

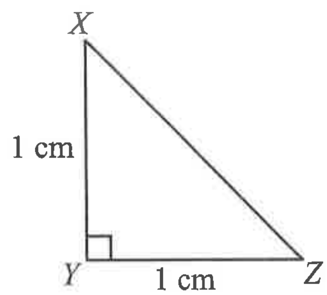
Not to scale



[Total 4 marks]

3 The diagram shows an isosceles triangle *XYZ* with a right angle at *Y*.

Use the triangle to prove that $\cos 45° = \frac{\sqrt{2}}{2}$.



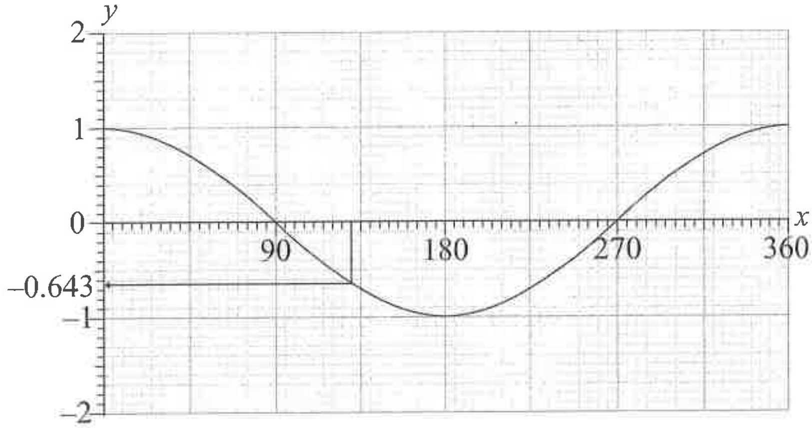
[Total 3 marks]

Score: ___ / 9

Solving Trig Equations in a Given Interval

1 The graph of $y = \cos x$ is shown below for $0° \le x \le 360°$.

As shown on the graph, $\cos 130° = -0.643$.

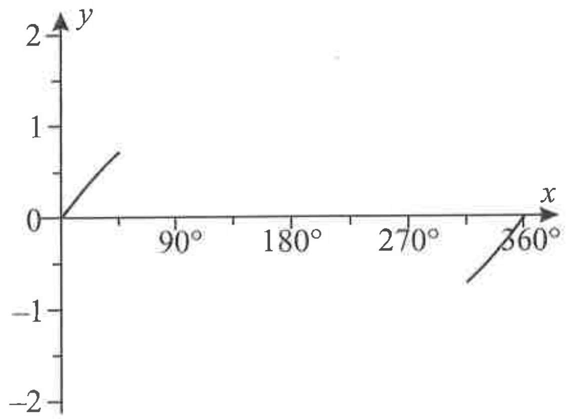


Give another value of x, found on this graph, where $\cos x = -0.643$.

$x =$ ....

[Total 1 mark]

2 $f(x) = \sin x$ for $0° \le x \le 360°$. Part of the graph of $y = f(x)$ is shown below.



a) Complete the diagram to show the graph of $y = f(x)$ for $0° \le x \le 360°$.

[2]

b) Write down the range of $f(x)$.

....

[1]

c) Write down the number of solutions each of these equations has in the interval $0° \le x \le 360°$.

$f(x) = 0.25$ ....

$f(x) = 1$ ....

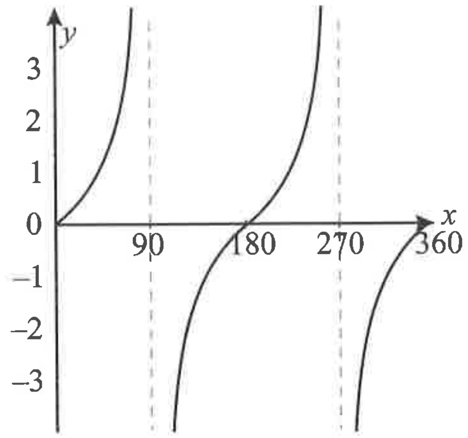
$f(x) = 0$ ....

$f(x) = -2$ ....

[2]

[Total 5 marks]

3 The diagram shows a sketch of $y = \tan x$ for $0° \leq x \leq 360°$.



You are given that $\tan 18° = 0.325$.

Write down the two solutions to the equation $\tan x = -0.325$ for $0° \leq x \leq 360°$.

$x =$ .... and $x =$ ....

[Total 2 marks]

4 Solve $\cos x = 0.65$ for $0° \leq x \leq 360°$. Give each solution to 1 decimal place.

....

[Total 3 marks]

5 Solve the equation $5 \sin^2 x - 2 = 0$ for $0° \leq x \leq 360°$.
Give your solutions to 1 decimal place.

....

[Total 5 marks]

Score: $\frac{\quad}{16}$

Trig Identities

1 Find all the values of x in the interval $0 \leq x \leq 360°$, for which $2 - \sin x = 2 \cos^2 x$.

....

[Total 6 marks]

2 For the equation $2(1 - \cos x) = 3 \sin^2 x$,

a) Show that the equation can be written as $3 \cos^2 x - 2 \cos x - 1 = 0$.

[2]

b) Use this to solve the equation for $0 \leq x \leq 360°$, giving your answers to 1 decimal place.

....

[4]

[Total 6 marks]

3 Prove the identity $\sin^2 \theta \tan \theta \equiv \tan \theta - \sin \theta \cos \theta$.

[Total 3 marks]

Score: ☐ / **15**

Candidate Surname	Candidate Forename(s)

Centre Number	Candidate Number	Candidate Signature

CGP

Further Mathematics Level 2

Paper 1 (Non-Calculator)

Practice Paper

Time allowed: 1 hour 30 minutes

You must have:
Pen, pencil, eraser, ruler, protractor, pair of compasses.
You may use tracing paper.

You are **not allowed** to use a calculator.

Instructions to candidates

- Use **black** ink to write your answers.
- Write your name and other details in the spaces provided above.
- Answer **all** questions in the spaces provided.
- In calculations show clearly how you worked out your answers.
- Do all rough work on the paper.

Information for candidates

- The marks available are given in brackets at the end of each question.
- You may get marks for method, even if your answer is incorrect.
- There are 15 questions in this paper. There are no blank pages.
- There are 70 marks available for this paper.

Get the answers online

Your free Online Edition of this book includes fully worked solutions
for this Exam Paper that you can print out.
There's more info about how to get your Online Edition at the front of this book.

Answer ALL the questions.

Write your answers in the spaces provided.

You must show all of your working.

1 Solve the equation $x(x + 2) + (x + 3)(x - 2) = (2x - 1)(x + 3)$

....

....

....

....

....

$x =$ ....

(4 marks)

2 Work out $\begin{pmatrix} -2 & 3 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} -3 & 1 \\ 1 & 2 \end{pmatrix}$.

Answer ....

(3 marks)

7

3 Simplify $\dfrac{x^{\frac{9}{2}} \times x^{\frac{3}{2}}}{\sqrt{x^8}}$.

....

....

....

....

Answer ....

(3 marks)

4 The operation ◊ is defined by the rule: $x \lozenge y = (x + y)^2 - 2x - 5$.
For example, $3 \lozenge 2 = (3 + 2)^2 - (2 \times 3) - 5 = 25 - 6 - 5 = 14$.

(a) Show that $\sqrt{2} \lozenge 1 = k$, where k is an integer to be found.

....

....

....

....

$k =$ ....

(3 marks)

(b) If x is a positive integer, show that $x \lozenge 3$ is always a square number.

....

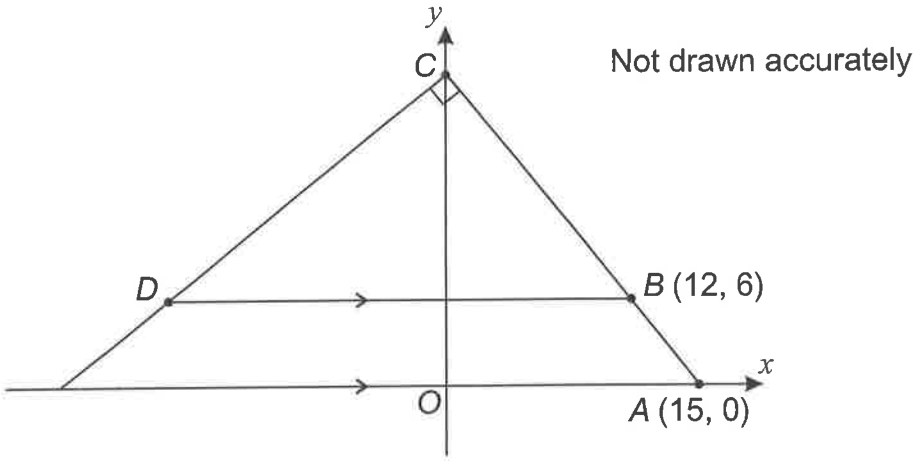
....

....

....

(3 marks)

5 C is the point on the y-axis such that A (15, 0), B (12, 6) and C lie on a straight line.



DC is perpendicular to CA and DB is horizontal.
Work out the coordinates of point D.

....

....

....

....

....

....

....

....

....

....

....

....

....

Answer (......................., ...)

(5 marks)

14

6 Rearrange the formula $3m = \dfrac{2m + 5n}{1 - n}$ to make n the subject.

....

....

....

....

....

n = ...

(4 marks)

7 Work out the value of $\left(\dfrac{y}{2x}\right)^2$ given that:

$3y = 4 - 2x$
$5y - 7x = 48$

....

....

....

....

....

....

....

....

....

Answer ....

(5 marks)

8 Solve the equation $\sqrt{32 - \sqrt{x}} = 5$.

....

....

....

....

$x =$ ....

(3 marks)

9 $y = (2x - 3)(x^2 + 1)$

(a) Find $\frac{dy}{dx}$.

....

....

....

....

$\frac{dy}{dx} =$ ....

(3 marks)

(b) Find the equation of the tangent to the curve $y = (2x - 3)(x^2 + 1)$ at the point (2, 5). Give your equation in the form $y = mx + c$.

....

....

....

....

Answer ....

(4 marks)

19

10 A is the point (–3, 12). B is the point (13, 0).

(a) Explain why the length of AB is 20 units.

....

....

....

....

(2 marks)

(b) Q is the point on AB such that $AQ : QB = 3 : 1$.
Find the equation of the circle with centre Q that passes through A.

....

....

....

....

....

....

....

....

Answer ....

(5 marks)

11 Simplify $\dfrac{6x^2 + 3xy}{x^2 - y^2} \div \dfrac{2x^2 + 3xy + y^2}{x - y}$.

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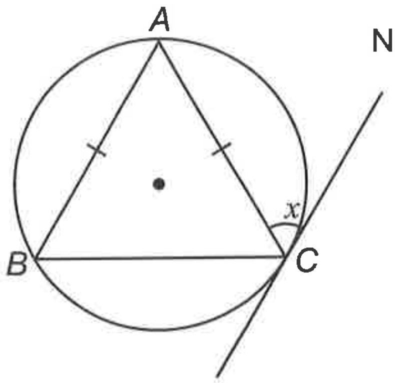
Answer ....

(5 marks)

12 The diagram shows a triangle ABC drawn inside a circle. $AB = AC$.
A tangent to the circle is drawn at C.



The angle between the tangent and AC is x.
If the tangent is parallel to AB, prove that ABC is an equilateral triangle.
Give geometrical reasons to support the statements you make.

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(4 marks)

16

13 (a) Express $2x^2 - 8x + 11$ in the form $a(x + b)^2 + c$.

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Answer ....

(4 marks)

(b) Write down the coordinates of the minimum point of the curve $y = 2x^2 - 8x + 11$.

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Answer ....

(1 mark)

(c) Write down the set of values of x for which $y = 2x^2 - 8x + 11$ is decreasing.

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Answer ....

(1 mark)

14 Solve the equation $2\sqrt{2} + y = \dfrac{1}{3 - 2\sqrt{2}}$ by rationalising the denominator.

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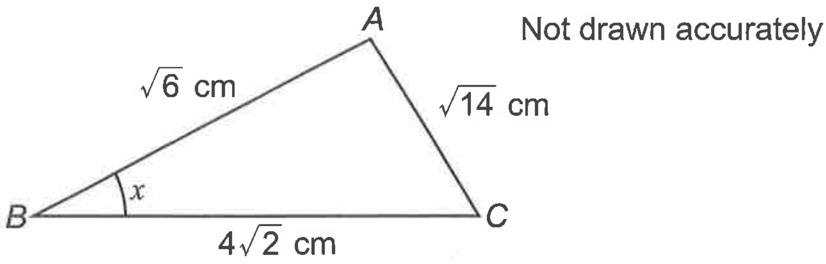
$y =$ ....

(4 marks)

15 The diagram shows a triangle *ABC*.



$AB = \sqrt{6}$ cm, $BC = 4\sqrt{2}$ cm, $AC = \sqrt{14}$ cm
Show that $x = 30°$.

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(4 marks)

14

Candidate Surname | Candidate Forename(s)

Centre Number | Candidate Number | Candidate Signature

CGP

Further Mathematics — Level 2

Paper 2 (Calculator)

Practice Paper

Time allowed: 2 hours

You must have:
Pen, pencil, eraser, ruler, protractor, pair of compasses.
You may use tracing paper.
You may use a calculator.

Instructions to candidates

- Use **black** ink to write your answers.
- Write your name and other details in the spaces provided above.
- Answer **all** questions in the spaces provided.
- In calculations show clearly how you worked out your answers.
- Do all rough work on the paper.

Information for candidates

- The marks available are given in brackets at the end of each question.
- You may get marks for method, even if your answer is incorrect.
- There are 22 questions in this paper. There are no blank pages.
- There are 105 marks available for this paper.

Get the answers online
Your free Online Edition of this book includes fully worked solutions for this Exam Paper that you can print out.
There's more info about how to get your Online Edition at the front of this book.

Answer ALL the questions.

Write your answers in the spaces provided.

You must show all of your working.

1 Solve $6(t - 3) \geq 12 - 2t$

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Answer ....

(3 marks)

2 The line segment joining the points (–3, –7) and (2, 8) has the same gradient as the line segment joining (–1, –9) to the point (3, c).

Work out the value of c.

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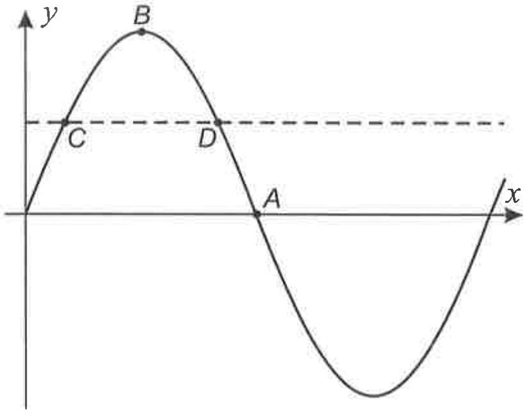
c = ...

(3 marks)

6

3 The diagram shows a sketch of part of the curve of $y = \sin x$.



Circle the correct answer to each of the following questions.

(a) The coordinates of A are

(90, 0) (180, 0) (360, 0)

(1 mark)

(b) The equation of the tangent at B is

$y = 1$ $y = 2$ $x = 90$ $x = 1$

(1 mark)

(c) If the x-coordinate of C is a, then the x-coordinate of D is

$90 - a$ $90 + a$ $180 - a$ $360 - a$

(1 mark)

4 A bag contains only red balls and yellow balls.

It has 6 more red balls than yellow balls. 4 red balls and 3 yellow balls are taken from the bag. The ratio of red balls to yellow balls remaining in the bag stays the same.

Calculate the total number of balls that were in the bag initially.

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Answer ....

(4 marks)

5 x and y are integers such that $0 < x < y$.

Write the following expressions in order of size, starting with the smallest.

$$\frac{x}{y}, \quad \frac{y}{x}, \quad \left(\frac{x}{y}\right)^2, \quad \left(\frac{y}{x}\right)^0$$

Answer ....

(3 marks)

6 n is a positive whole number.

Prove that $(2n - 1)(4n^2 + 2n + 1) + 12n + 5$ is a multiple of 4.

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(3 marks)

13

7 Factorise fully:

(a) $8m^6 - 18m^2n^2$

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Answer ....

(3 marks)

(b) $5(x - 2y) - 3(2y - x)(3x + 1)$

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Answer ....

(3 marks)

8 Describe fully the single transformation represented by the matrix $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$.

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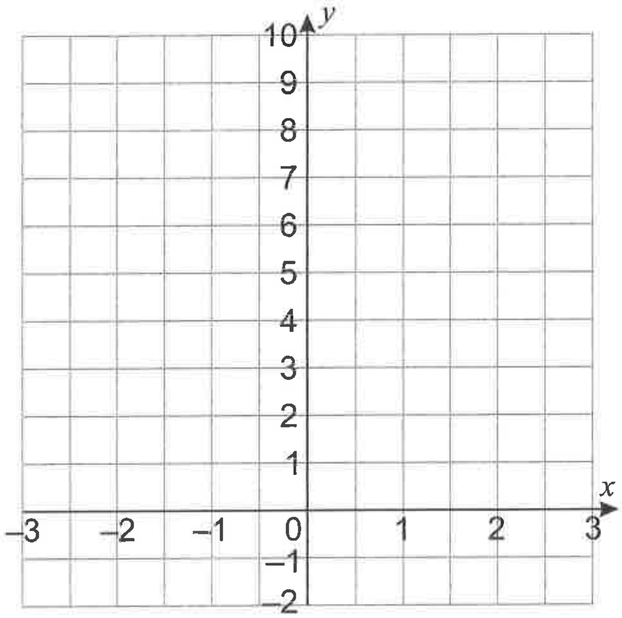
(3 marks)

9 A function f(x) has domain $-3 \le x \le 3$.

It is defined as:

$$f(x) = \begin{cases} 3x + 9 & -3 \le x < 0 \\ 9 - x^2 & 0 \le x < 2 \\ 5 & 2 \le x \le 3 \end{cases}$$

(a) Draw the graph of $y = f(x)$.



(3 marks)

(b) Write down the range of f(x).

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Answer ....

(2 marks)

(c) Solve the equation f(x) = 2.

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Answer ....

(2 marks)

16

10 Some of the terms of a quadratic sequence are shown in the table.

1st term	2nd term	3rd term	4th term	5th term
3		5	9	15

(a) Work out the second term in the sequence.

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Answer ....

(2 marks)

(b) Find an expression for the nth term of the sequence.

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Answer ....

(4 marks)

11 Solve the simultaneous equations

$y - 2x + 5 = 0$ and $x^2 + y^2 - 3x - 13 = 0$

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Answer ....

(5 marks)

12 A curve has equation $y = ax^4 - 5ax^2 + 2$.
The rate of change of y with respect to x when $x = -2$ is -36.
Work out the value of a.

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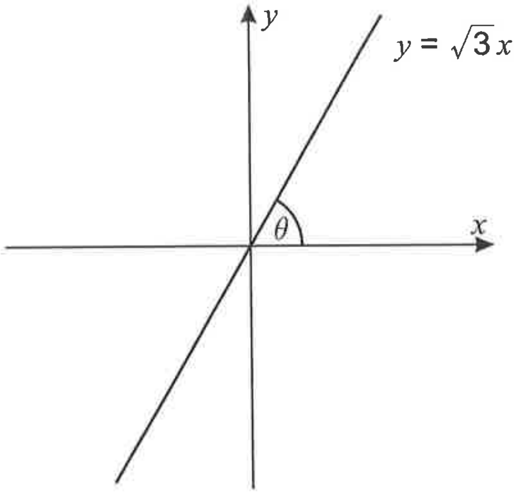
$a =$ ....

(4 marks)

15

13 (a) The diagram shows the line $y = \sqrt{3}x$.



Show that $\theta = 60°$.

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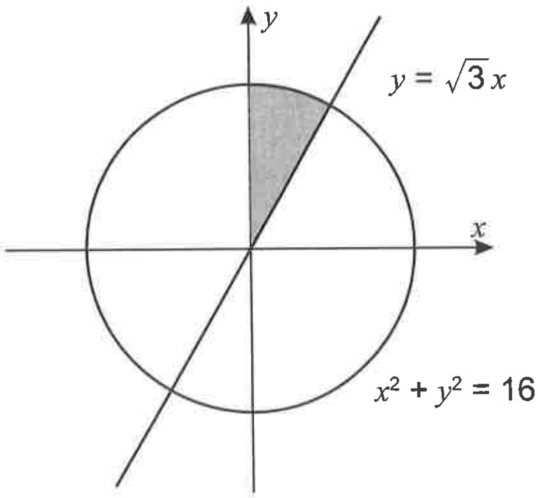
(2 marks)

(b) The diagram shows the circle $x^2 + y^2 = 16$.



The sector enclosed by the circle and the line $y = \sqrt{3}x$ is shaded.
Work out the area of the shaded sector. Give your answer in terms of π.

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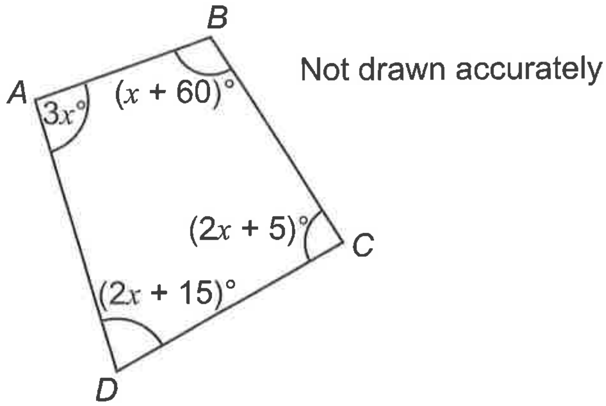
Answer ....

(3 marks)

14 The diagram shows a quadrilateral *ABCD*.



Prove that the quadrilateral is cyclic.

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(4 marks)

9

15 $g(x) = \frac{2x+1}{x}$

Prove the equation $g(x) - g(x + 2) = -2$

has a single solution $x = k$, where k is an integer to be found.

(7 marks)

16 Show that $\frac{\tan\theta(1-\sin^2\theta)}{\cos\theta}$ simplifies to $\sin\theta$.

(3 marks)

17 (a) Find the coordinates of the stationary points of the curve $y = 2x^3 - 3x^2 + 4$.

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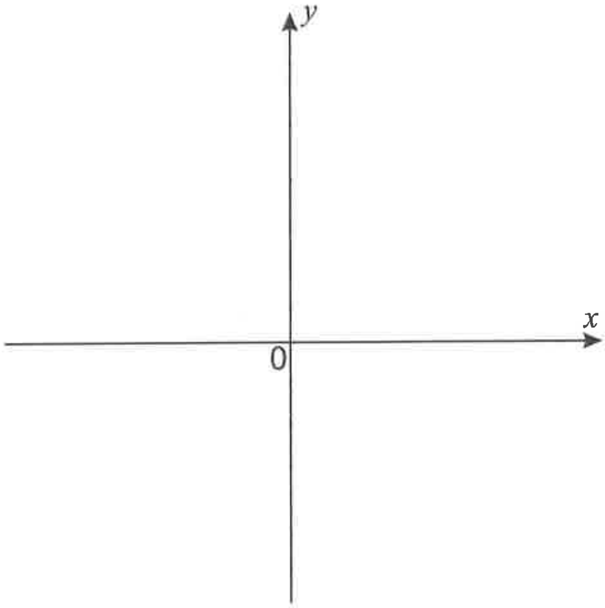
Answer ....

(5 marks)

(b) Sketch the curve $y = 2x^3 - 3x^2 + 4$.



(2 marks)

(c) Work out the number of solutions to the equation $2x^3 - 3x^2 + 1 = 0$.

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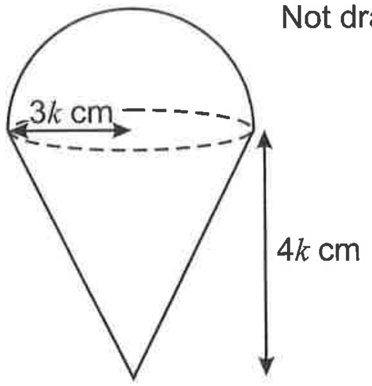
Answer ....

(2 marks)

19

18 The diagram shows a solid object made up of a hemisphere of radius $3k$ cm and a cone with vertical height $4k$ cm.



The total surface area of the object is 3993π cm^2.

Work out the value of k.

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k = ...

(6 marks)

19 Matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$.

The point $P(x, y)$ is acted upon first by the transformation that corresponds to matrix **A** and then the transformation that corresponds to matrix **B**.

The image of P after the two transformations is $P'(-10, 5)$.

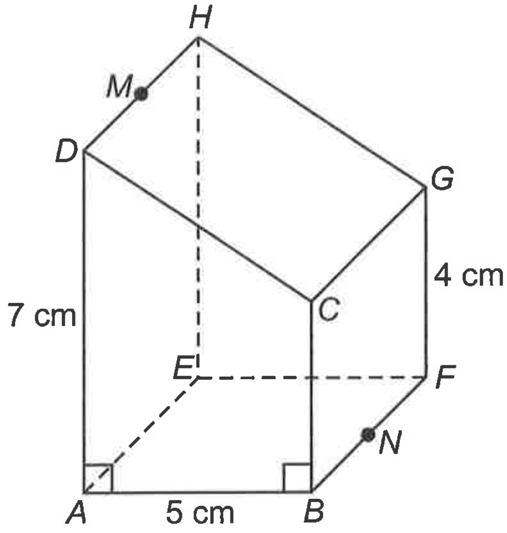
Find the coordinates of P.

Answer ....

(5 marks)

20 The diagram shows a prism *ABCDEFGH*.

ABFE is a square, and *M* and *N* are the midpoints of *DH* and *BF* respectively.



(a) Calculate the angle between the line *HB* and the base *ABFE*.
Give your answer to 3 significant figures.

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Answer ....

(3 marks)

(b) Calculate angle *BMF*. Give your answer to 3 significant figures.

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Answer ....

(3 marks)

17

21 $f(x) = x^3 + ax^2 - 14x + b$.

$(x + 2)$ and $(x - 4)$ are both factors of $f(x)$.

Work out the values of a and b.

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a = ..., b = ...

(5 marks)

22 Solve the equation $9\cos^2 x - 5\cos x = 0$ for $0 \leq x \leq 360°$.
Give your answers to 3 significant figures where appropriate.

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Answer ....

(5 marks)

10

Answers

Section One — Number

Page 4: Fractions

1 a) $4\frac{2}{5} + 3\frac{1}{4} = \frac{22}{5} + \frac{13}{4} = \frac{88}{20} + \frac{65}{20} = \frac{88+65}{20} = \frac{153}{20}$ or $7\frac{13}{20}$

[3 marks available — 1 mark for writing as improper fractions, 1 mark for writing over a common denominator, 1 mark for the correct answer]

b) $2\frac{5}{6} - 1\frac{1}{5} = \frac{17}{6} - \frac{6}{5} = \frac{85}{30} - \frac{36}{30} = \frac{85-36}{30} = \frac{49}{30}$ or $1\frac{19}{30}$

[3 marks available — 1 mark for writing as improper fractions, 1 mark for writing over a common denominator, 1 mark for the correct answer]

If you've used a different method in Q1, but still shown your working, and ended up with the same final answer, then you still get full marks.

2 a) $4\frac{3}{5} \times 2\frac{1}{3} = \frac{23}{5} \times \frac{7}{3} = \frac{23 \times 7}{5 \times 3}$ ***[1 mark]***

$= \frac{161}{15}$ or $10\frac{11}{15}$ ***[1 mark]***

[2 marks available in total — as above]

b) $4\frac{1}{3} \div 2\frac{3}{5} = \frac{13}{3} \div \frac{13}{5} = \frac{13}{3} \times \frac{5}{13} = \frac{1}{3} \times \frac{5}{1} = \frac{5}{3}$ or $1\frac{2}{3}$

[3 marks available — 1 mark for taking the reciprocal and multiplying the two fractions together, 1 mark for an equivalent fraction, 1 mark for the correct final answer]

3 First work out the multiplication:

$2\frac{1}{4} \times 2\frac{1}{3} = \frac{9}{4} \times \frac{7}{3} = \frac{21}{4}$

Then do the subtraction:

$7\frac{1}{5} - \frac{21}{4} = \frac{36}{5} - \frac{21}{4} = \frac{144}{20} - \frac{105}{20} = \frac{39}{20}$ or $1\frac{19}{20}$

[5 marks available — 1 mark for attempting the multiplication before the subtraction, 1 mark for writing the two fractions to be multiplied as improper fractions, 1 mark for correctly working out the multiplication, 1 mark for writing the fractions to be subtracted over a common denominator, 1 mark for a correct final answer]

4 Area of a triangle = $\frac{1}{2}$ × base × height

Base (AD) = $6\frac{3}{4} - 2\frac{7}{8} = \frac{27}{4} - \frac{23}{8} = \frac{31}{8}$

Height (BD) = $6\frac{3}{4} \times \frac{2}{3} = \frac{27}{4} \times \frac{2}{3} = \frac{9}{2}$

Area = $\frac{1}{2} \times \frac{31}{8} \times \frac{9}{2} = \frac{31 \times 9}{2 \times 8 \times 2} = \frac{279}{32}$ in^2 or $8\frac{23}{32}$ in^2

[5 marks available — 1 mark for a correct method to find the base length, 1 mark for finding AD, 1 mark for a correct method to find the height, 1 mark for correctly finding BD and 1 mark for a correct final answer]

Page 5: Fractions, Decimals and Percentages

1 a) $\frac{3}{5} = \frac{6}{10} = 0.6$ ***[1 mark]***

b) 0.04 × 100 = 4% ***[1 mark]***

c) $65\% = \frac{65}{100}$

$= \frac{65 \div 5}{100 \div 5} = \frac{13}{20}$ ***[1 mark]***

2 a) Divide the top and bottom of the fraction by 8: $\frac{56}{80} = \frac{7}{10}$

7 ÷ 10 = 0.7 ***[1 mark]***

0.7 × 100 = 70% ***[1 mark]***

[2 marks available in total — as above]

Or you could convert the fraction to one with a denominator of 100: $\frac{7}{10} = \frac{70}{100} = 70\%$.

b) $90\% = \frac{90}{100}$ ***[1 mark]***

$= \frac{90 \div 10}{100 \div 10} = \frac{9}{10} = \frac{9 \times 8}{10 \times 8} = \frac{72}{80}$ ***[1 mark]***

[2 marks available in total — as above]

3 Convert $\frac{8}{25}$ to a percentage:

$\frac{8}{25} = \frac{8 \times 4}{25 \times 4} = \frac{32}{100} = 32\%$ ***[1 mark]***

100% – (42% + 32%) = 100% – 74% = 26% ***[1 mark]***

So 26% of the blocks are green.

100% = 150, 10% = 15, 1% = 1.5

26% = 20% + 6%

= (2 × 15) + (6 × 1.5)

= 30 + 9 = 39 blocks are green. ***[1 mark]***

[3 marks available in total — as above]

Page 6: Percentages

1 a) 24% of round counters are black

(2 ÷ 5) × 100 = 40% of square counters are black

(18 ÷ 48) × 100 = 37.5% of triangular counters are black

Therefore the square counters have the highest percentage of black counters.

[3 marks available — 1 mark for calculating percentage of square, 1 mark for calculating percentage of triangular, 1 mark for square as answer]

b) Number of round counters which are black

= 0.24 × 75 = 18

Number of square counters which are black

= 0.4 × 65 = 26

Number of triangular counters which are black

= 18

Total number of counters = 75 + 65 + 48 = 188

(18 + 26 + 18) ÷ 188 × 100 = 32.978... = 32.98%

[4 marks available — 1 mark for calculating the number of round and square counters which are black, 1 mark for finding the total number of counters, 1 mark for calculating percentage of total, 1 mark for correct answer]

2 13 104 = 117%

13 104 ÷ 117 = 112 = 1% ***[1 mark]***

112 × 100 = 100% ***[1 mark]***

= 11 200 ***[1 mark]***

[3 marks available in total — as above]

3 $x = 1.25s$ ***[1 mark]***

$y = 0.64t$ ***[1 mark]***

$x = \frac{1}{y}$ means that $1.25s = \frac{1}{0.64t}$ ***[1 mark]***

Rearrange to give $st = \frac{1}{1.25 \times 0.64} = 1.25$ ***[1 mark]***

[4 marks available in total — as above]

4 15% of a is $0.15a$

The number of sweets left in bag A is $0.85a$. ***[1 mark]***

The number of sweets then in bag B is $b + 0.15a$. ***[1 mark]***

$0.85a = b + 0.15a$ ***[1 mark]***

$0.7a = b$, so b is 70% of a. ***[1 mark]***

[4 marks available in total — as above]

Pages 7-8: Ratios

1 84 ÷ (3 + 5 + 4 + 8) = 84 ÷ 20 ***[1 mark]***

= 4.2

Largest share is 8 × 4.2 ***[1 mark]***

= 33.6 ***[1 mark]***

[3 marks available in total — as above]

2 Chloe, André, Nasir and Simone shared the money in the ratio 1 : 2 : 4 : 8
[1 mark for 1 : 2 : 4 : 8 or an equivalent ratio (order may be different)]
$660 \div (1 + 2 + 4 + 8) = 44$ ***[1 mark]***
Simone got £44 × 8 = £352 ***[1 mark]***
[3 marks available in total — as above]
You could answer this question using a formula — if you let x be the amount of money that Chloe gets, then x + 2x + 4x + 8x = £660.

3 $\frac{2y}{5} = \frac{2}{5} \times 35\,000 = 14\,000$ ***[1 mark]***
$14\,000 \div (11 + 17) = 14\,000 \div 28 = 500$ ***[1 mark]***
Larger amount = 500 × 17 = 8500 ***[1 mark]***
[3 marks available in total — as above]

4 Substituting $x = 0$ into $(4x - 3y)(2x^3 - 4y)$ gives $12y^2$
Substituting $x = 0$ into $(y + 6x)(9x + 8y)$ gives $8y^2$
So simplify the ratio $12y^2 : 8y^2$ to give 3 : 2
[2 marks available — 1 mark for correctly substituting x = 0 into the two expressions and 1 mark for a correctly simplified ratio]

5 Ratio of red : green is 1 : 3 = 4 : 12 and ratio of green : white is 4 : 3 = 12 : 9 ***[1 mark]***
So ratio of red : green : white = 4 : 12 : 9 ***[1 mark]***
The fraction of the tiles that are white is $\frac{9}{4 + 12 + 9} = \frac{9}{25}$ ***[1 mark]***
[3 marks available in total — as above]

6 $b - a$ is represented by 7 – 2 = 5 parts in the ratio,
so 5 parts = 40. ***[1 mark]***
Therefore 1 part = 40 ÷ 5 = 8. ***[1 mark]***
$a + b = 7 + 2 = 9$ parts, so $a + b = 9 \times 8 = 72$ ***[1 mark]***
[3 marks available in total — as above]
Alternatively, you could write $b = \frac{7}{2}a$, and then substitute this into the equation in the question, find values for a and b, and then work out the value of a + b.

7 $3250 \div (3 + 7) = 3250 \div 10 = 325$ ***[1 mark]***
$s = 325 \times 7 = 2275$ and $f = 325 \times 3 = 975$ ***[1 mark for both]***
So, $fx + 175s = 651\,625$ gives:
$975x + (175 \times 2275) = 651\,625$ ***[1 mark]***
$975x = 651\,625 - 398\,125 = 253\,500$ ***[1 mark]***
$x = 253\,500 \div 975 = 260$ ***[1 mark]***
[5 marks available in total — as above]

8 a) $h = \frac{5}{2}g$ ***[1 mark]***

b) In 10 years' time, the ages of Georgia and Harry will be $(g + 10)$ and $(h + 10)$. ***[1 mark]***
$(g + 10) : (h + 10) = 3 : 5$
So $(h + 10) = \frac{5}{3}(g + 10)$ ***[1 mark]***
Substitute in $\frac{5}{2}g$ for h (from part a):
$\frac{5}{2}g + 10 = \frac{5}{3}(g + 10)$ ***[1 mark]***
Solving for g gives $g = 8$ so Georgia is 8 years old ***[1 mark]***
[4 marks available in total — as above]

Section Two — Algebra

Page 9: Powers and Roots

1 a) $5a^{\frac{3}{2}} \times 4a^{\frac{5}{2}}b^3 = (5 \times 4) \times (a^{\frac{3}{2}} \times a^{\frac{5}{2}}) \times b^3 = 20a^4b^3$
[2 marks available — 1 mark for correct working, 1 mark for the correct answer]

b) $\frac{6a^{\frac{5}{2}}b^6}{3a^{\frac{1}{2}}b^4} = (6 \div 3) \times (a^{\frac{5}{2}} \div a^{\frac{1}{2}}) \times (b^6 \div b^4) = 2a^2b^2$
[2 marks available — 1 mark for correct working, 1 mark for the correct answer]

2 a) $16^{\frac{3}{2}} = (16^{\frac{1}{2}})^3 = (\sqrt{16})^3 = 4^3 = 64$
[2 marks available — 1 mark for finding the square root of 16, 1 mark for then finding 4 cubed]
With fractional powers, you can do the two steps in whichever order you want — it's usually easier to find the root first though, as long as it gives a nice number.

b) $64^6 = (16^{\frac{3}{2}})^6$ ***[1 mark]***
$= 16^9$, so $x = 9$ ***[1 mark]***
[2 marks available in total — as above]

3 $(4a^6)^{\frac{1}{2}} = \sqrt{4a^6} = 2a^3$ ***[1 mark]***
$\frac{5a^3b^5}{10a^5b^2} = \frac{5}{10} \times \frac{a^3}{a^5} \times \frac{b^5}{b^2} = \frac{1}{2} \times \frac{1}{a^2} \times b^3 = \frac{b^3}{2a^2}$ ***[1 mark]***
so $(4a^6)^{\frac{1}{2}} \times \frac{5a^3b^5}{10a^5b^2} = 2a^3 \times \frac{b^3}{2a^2} = ab^3$ ***[1 mark]***
[3 marks available in total — as above]

4 $\frac{x + 5x^3}{\sqrt{x}} = x^{-\frac{1}{2}}(x + 5x^3)$ ***[1 mark]***
$= x^{\frac{1}{2}} + 5x^{\frac{5}{2}}$ ***[1 mark]***
[2 marks available in total — as above]

Pages 10-11: Expanding Brackets

1 a) $4x(2x - 3) = (4x \times 2x) + (4x \times -3) = 8x^2 - 12x$
[2 marks available — 2 marks for both terms correct, otherwise 1 mark for one term correct]

b) $5a(3a + 6ab) = (5a \times 3a) + (5a \times 6ab) = 15a^2 + 30a^2b$
[2 marks available — 2 marks for both terms correct, otherwise 1 mark for one term correct]

c) $3p^3(8 - p^2) - 4p(2p^2 - 7p)$
$= [(3p^3 \times 8) + (3p^3 \times -p^2)] - [(4p \times 2p^2) + (4p \times -7p)]$
$= 24p^3 - 3p^5 - 8p^3 + 28p^2$
$= 16p^3 - 3p^5 + 28p^2$
[3 marks available — 3 marks for all three terms correct, otherwise 2 marks for two terms correct, otherwise 1 mark for one term correct]

2 $4a^2(2a - 5) + a(3a + 4a^2)$
$= [(4a^2 \times 2a) + (4a^2 \times -5)] + [(a \times 3a) + (a \times 4a^2)]$
$= 8a^3 - 20a^2 + 3a^2 + 4a^3$ ***[1 mark]***
$= 12a^3 - 17a^2$ ***[1 mark]***
[2 marks available in total — as above]

3 a) $(4t - 3)(2t + 5) = (4t \times 2t) + (4t \times 5) + (-3 \times 2t) + (-3 \times 5)$
$= 8t^2 + 20t - 6t - 15$
$= 8t^2 + 14t - 15$
[3 marks available — 3 marks for the correct answer, otherwise 1 mark for an unsimplified expansion with at least 2 terms correct, 1 mark for a fully correct unsimplified expansion]

b) $(2x + 9)^2 = (2x + 9)(2x + 9)$
$= (2x \times 2x) + (2x \times 9) + (9 \times 2x) + (9 \times 9)$
$= 4x^2 + 18x + 18x + 81$
$= 4x^2 + 36x + 81$
[3 marks available — 3 marks for the correct answer, otherwise 1 mark for an unsimplified expansion with at least 2 terms correct, 1 mark for a fully correct unsimplified expansion]

4 Area = ½ × base × height
$= \frac{1}{2} \times (2x + 6) \times (x - 1) = \frac{1}{2}(2x + 6)(x - 1)$ *[1 mark]*
$= \frac{1}{2} \times [(2x \times x) + (2x \times -1) + (6 \times x) + (6 \times -1)]$
$= \frac{1}{2} \times (2x^2 - 2x + 6x - 6)$
$= \frac{1}{2} \times (2x^2 + 4x - 6)$ *[1 mark]*
$= x^2 + 2x - 3$ *[1 mark]*
[3 marks available in total — as above]
You could also have multiplied (2x + 6) by ½ first of all. The area would then just be (x + 3)(x − 1), which is a bit simpler to multiply out.

5 a) $(4m - 3n)(3m + n)$
$= (4m \times 3m) + (4m \times n) + (-3n \times 3m) + (-3n \times n)$
$= 12m^2 + 4mn - 9mn - 3n^2$
$= 12m^2 - 5mn - 3n^2$
[3 marks available — 3 marks for the correct answer, otherwise 1 mark for an unsimplified expansion with at least 2 terms correct, 1 mark for a fully correct unsimplified expansion]

b) $(n - 3)(n - 3) = n^2 - 3n - 3n + 9 = n^2 - 6n + 9$ *[1 mark]*
$(n - 3)^3 = (n - 3)(n^2 - 6n + 9)$
$= (n \times n^2) + (n \times -6n) + (n \times 9) + (-3 \times n^2) + (-3 \times -6n) + (-3 \times 9)$
$= n^3 - 6n^2 + 9n - 3n^2 + 18n - 27$ *[1 mark]*
$= n^3 - 9n^2 + 27n - 27$ *[1 mark]*
[3 marks available in total — as above]

6 $(2x + 3)(x^2 - 2x + 2)$
$= (2x \times x^2) + (2x \times -2x) + (2x \times 2) + (3 \times x^2) + (3 \times -2x) + (3 \times 2)$
$= 2x^3 - 4x^2 + 4x + 3x^2 - 6x + 6$
$= 2x^3 - x^2 - 2x + 6$
[3 marks available — 1 mark for correctly multiplying the second bracket by 2x, 1 mark for correctly multiplying the second bracket by 3, 1 mark for simplifying to give the correct final answer]

7 $(x + 1)(x + 2) = x^2 + 2x + x + 2 = x^2 + 3x + 2$ *[1 mark]*
$(3x + 2)(x^2 + 3x + 2) = (3x \times x^2) + (3x \times 3x) + (3x \times 2) + (2 \times x^2) + (2 \times 3x) + (2 \times 2)$
$= 3x^3 + 9x^2 + 6x + 2x^2 + 6x + 4$ *[1 mark]*
$= 3x^3 + 11x^2 + 12x + 4$ *[1 mark]*
$3x(x^2 - 2x + 4) = (3x \times x^2) + (3x \times -2x) + (3x \times 4)$
$= 3x^3 - 6x^2 + 12x$ *[1 mark]*
So $(3x + 2)(x + 1)(x + 2) - 3x(x^2 - 2x + 4)$
$= 3x^3 + 11x^2 + 12x + 4 - 3x^3 + 6x^2 - 12x$
$= 17x^2 + 4$ *[1 mark]*
[5 marks available in total — as above]
You could have started by multiplying a different pair of brackets together, e.g. (3x + 2) by (x + 1).

Pages 12-13: Factorising

1 $8a^2 - 48ab = 8a(a - 6b)$ *[1 mark]*

2 a) $16x + 4x^2 = 4x(4 + x)$ *[1 mark]*

b) $25y^2 - 40y^3 = 5(5y^2 - 8y^3)$
$= 5y^2(5 - 8y)$
[2 marks available — 2 marks for a complete factorisation, otherwise 1 mark for a partial factorisation]

c) $6v^2w^3 + 30v^4w^2 = 6(v^2w^3 + 5v^4w^2)$
$= 6v^2w^2(w + 5v^2)$
[2 marks available — 2 marks for a complete factorisation, otherwise 1 mark for a partial factorisation]

3 a) $4x^2 - 25 = (2x)^2 - 5^2 = (2x + 5)(2x - 5)$
[2 marks available — 2 marks for both factors correct, otherwise 1 mark for one factor correct]

b) $72n^4 - 200m^2 = 8(9n^4 - 25m^2) = 8[(3n^2)^2 - (5m)^2]$
$= 8(3n^2 + 5m)(3n^2 - 5m)$
[3 marks available — 3 marks for all three factors correct, otherwise 2 marks for two factors correct, otherwise 1 mark for one factor correct]

4 a) $16a^4b^2 - 49c^6 = (4a^2b)^2 - (7c^3)^2$
$= (4a^2b + 7c^3)(4a^2b - 7c^3)$
[2 marks available — 2 marks for the correct answer, otherwise 1 mark for recognising a difference of two squares]

b) $8x^3 - 18xy^2 = 2x(4x^2 - 9y^2)$ *[1 mark]*
$= 2x((2x)^2 - (3y)^2)$ *[1 mark]*
$= 2x(2x + 3y)(2x - 3y)$ *[1 mark]*
[3 marks available in total — as above]

5 $(2x + 3y)^3 + (2x + 3y)^2(x - y) = (2x + 3y)^2[(2x + 3y) + (x - y)]$ *[1 mark]*
$= (2x + 3y)^2(3x + 2y)$ *[1 mark]*
[2 marks available in total — as above]

6 $(3x + y)^2 - (x + y)^2 = [(3x + y) + (x + y)][(3x + y) - (x + y)]$ *[1 mark]*
$= (4x + 2y)(2x)$ *[1 mark]*
$= 4x(2x + y)$ *[1 mark]*
[3 marks available in total — as above]
This could be answered by squaring each of the brackets, subtracting the expansions and then factorising.

7 $(4x^2 - 1) = (2x + 1)(2x - 1)$ *[1 mark for each factor]*
$(4x^2 - 1) - (2x - 1)(x + 1) = (2x + 1)(2x - 1) - (2x - 1)(x + 1)$
$= (2x - 1)[(2x + 1) - (x + 1)]$ *[1 mark]*
$= x(2x - 1)$ *[1 mark]*
[4 marks available in total — as above]

Page 14: Manipulating Surds

1 $(6 + \sqrt{2})(1 - \sqrt{2}) = (6 \times 1) + (6 \times -\sqrt{2}) + (\sqrt{2} \times 1) + (\sqrt{2} \times -\sqrt{2})$
$= 6 - 6\sqrt{2} + \sqrt{2} - 2$
$= 4 - 5\sqrt{2}$
[2 marks available — 1 mark for correct working, 1 mark for the correct answer]

2 Multiply top and bottom by $(3 + \sqrt{5})$ to 'rationalise the denominator':
$\frac{5 + \sqrt{5}}{3 - \sqrt{5}} = \frac{(5 + \sqrt{5})(3 + \sqrt{5})}{(3 - \sqrt{5})(3 + \sqrt{5})}$ *[1 mark]*
$= \frac{15 + 5\sqrt{5} + 3\sqrt{5} + 5}{9 - 5}$
[1 mark for numerator, 1 mark for denominator]
$= \frac{20 + 8\sqrt{5}}{4}$ *[1 mark]*
$= 5 + 2\sqrt{5}$ *[1 mark]*
[5 marks available in total — as above]

3 $2\sqrt{175} = 2\sqrt{25 \times 7} = 2 \times 5\sqrt{7} = 10\sqrt{7}$
$\sqrt{28} = \sqrt{4 \times 7} = 2\sqrt{7}$
So $2\sqrt{175} - \sqrt{28} = 10\sqrt{7} - 2\sqrt{7} = 8\sqrt{7}$
[2 marks available — 2 marks for the correct answer, otherwise 1 mark for correctly simplifying either surd]

4 Multiply numerators and denominators on both sides by $\sqrt{3}$:
$a = \frac{\sqrt{3} \times \sqrt{3}}{2 + \sqrt{3}} = \frac{3}{2 + \sqrt{3}}$ *[1 mark]*
$= \frac{3(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})}$ *[1 mark]*
$= \frac{6 - 3\sqrt{3}}{4 - 3}$ *[1 mark]*
$= 6 - 3\sqrt{3}$ *[1 mark]*
[4 marks available in total — as above]

Page 15: Solving Equations

1 a) $7b - 5 = 3(b + 1)$
$7b - 5 = 3b + 3$
$7b - 3b = 3 + 5$
$4b = 8$
$b = \frac{8}{4} = 2$
[3 marks available — 1 mark for expanding out the bracket, 1 mark for adding 5 and subtracting 3b from each side, 1 mark for the correct answer]

b) $\frac{45 - z}{5} = 6$
$45 - z = 30$
$z = 45 - 30$
$z = 15$
[2 marks available — 1 mark for multiplying both sides by 5, 1 mark for correct answer]

2 $\frac{5 - 3x}{2} + \frac{6x + 1}{5} = 15$
$\frac{2 \times 5 \times (5 - 3x)}{2} + \frac{2 \times 5 \times (6x + 1)}{5} = 150$
$5(5 - 3x) + 2(6x + 1) = 150$ *[1 mark]*
$25 - 15x + 12x + 2 = 150$
$-15x + 12x = 150 - 25 - 2$
$-3x = 123$ *[1 mark]*
$x = -41$ *[1 mark]*
[3 marks available in total — as above]

3 The perimeter is $(3x - 4) + (x + 3) + (14 - x) + (4x + 5)$, so...
$(3x - 4) + (x + 3) + (14 - x) + (4x + 5) = 67$ *[1 mark]*
$7x + 18 = 67$ *[1 mark]*
$7x = 67 - 18$
$7x = 49$
$x = 7$ *[1 mark]*
[3 marks available in total — as above]

Pages 16-17: Rearranging Formulas

1 a) $y = \frac{x - 6}{4}$, so $4y = x - 6$ and $x = 4y + 6$
[2 marks available — 1 mark for multiplying both sides by 4, 1 mark for the correct answer]

b) When $y = 11$, $x = (4 \times 11) + 6 = 44 + 6 = 50$
[2 marks available — 1 mark for correct substitution, 1 mark for the correct answer]

2 $s = \frac{1}{4}g^2t^2$, so $g^2t^2 = 4s$ *[1 mark]*
$t^2 = \frac{4s}{g^2}$ *[1 mark]*,
$t = \pm\sqrt{\frac{4s}{g^2}} = \frac{\pm 2\sqrt{s}}{g}$ *[1 mark]*
[3 marks available in total — as above]

3 a) $a - y = \frac{2b + y}{3a}$, so...
$3a(a - y) = 2b + y$ *[1 mark]*
$3a^2 - 3ay = 2b + y$
$y + 3ay = 3a^2 - 2b$ *[1 mark]*,
$y(1 + 3a) = 3a^2 - 2b$ *[1 mark]*,
$y = \frac{3a^2 - 2b}{1 + 3a}$ *[1 mark]*
[4 marks available in total — as above]

b) When $a = 3$ and $b = 4$, $y = \frac{3(3^2) - 2(4)}{1 + 3(3)} = \frac{19}{10}$ or 1.9
[2 marks available — 1 mark for correct substitution, 1 mark for the correct answer]

4 $x = \sqrt{\frac{(2 + n)}{(1 - 3n)}}$
$x^2 = \frac{2 + n}{1 - 3n}$ *[1 mark]*
$x^2(1 - 3n) = 2 + n$
$x^2 - 3x^2n = 2 + n$ *[1 mark]*
$3x^2n + n = x^2 - 2$ *[1 mark]*
$n(3x^2 + 1) = x^2 - 2$ *[1 mark]*
$n = \frac{x^2 - 2}{3x^2 + 1}$ *[1 mark]*
[5 marks available in total — as above]

5 $\frac{W}{Q} = \frac{h - c}{h}$
$hW = Q(h - c)$ *[1 mark]*
$Qc = Qh - hW$ *[1 mark]*
$Qc = h(Q - W)$ *[1 mark]*
$h = \frac{Qc}{Q - W}$ *[1 mark]*
[4 marks available in total — as above]

6 $2ay - 5 = 7a + 2y$
$2ay - 7a = 2y + 5$ *[1 mark]*
$a(2y - 7) = 2y + 5$ *[1 mark]*
$a = \frac{2y + 5}{2y - 7}$ *[1 mark]*
[3 marks available in total — as above]

7 $2y = 5 - \frac{\sqrt[3]{r}}{t}$
$\frac{\sqrt[3]{r}}{t} = 5 - 2y$ *[1 mark]*
$\sqrt[3]{r} = 5t - 2ty$ *[1 mark]*
$r = (5t - 2ty)^3$ *[1 mark]*
[3 marks available in total — as above]

8 There are x counters in the bag, then $\frac{x}{2}$ counters are added to the bag and y counters are removed. So
$x + \frac{x}{2} - y = 15$ *[1 mark]*
$\frac{3}{2}x = 15 + y$ *[1 mark]*
$x = 10 + \frac{2y}{3}$ *[1 mark]*
[3 marks available in total — as above]

Pages 18-19: Factorising Quadratics

1 $(x + 5)(x + 7)$
[2 marks available — 1 mark for correct numbers in brackets, 1 mark for correct signs]
The brackets can be either way around — $(x + 7)(x + 5)$ is also correct.

2 a) $(y - 4)(y + 6)$
[2 marks available — 1 mark for correct numbers in brackets, 1 mark for correct signs]

b) $(3x + 2)(x - 5)$
[2 marks available — 1 mark for correct numbers in brackets, 1 mark for correct signs]

3 a) $(x - 5)(x - 6)$
[2 marks available — 1 mark for correct numbers in brackets, 1 mark for correct signs]

b) $x - 5 = 0$ or $x - 6 = 0$
$x = 5$ or $x = 6$
[1 mark for both solutions correct]

4 $x^2 + 6x - 27 = 0$
$(x + 9)(x - 3) = 0$
[1 mark for correct numbers in brackets, 1 mark for correct signs]
$x + 9 = 0$ or $x - 3 = 0$
$x = -9$ or $x = 3$
[1 mark for both solutions]
[3 marks available in total — as above]

5 $4n^2 + 16n + 15 = (2n + 3)(2n + 5)$
[2 marks available — 1 mark for correct numbers in brackets, 1 mark for correct signs]

6 $6t^2 - 7t - 20$ factorises to $(2t - 5)(3t + 4)$
So the factorisation of $6t^2 - 7tu - 20u^2$ is $(2t - 5u)(3t + 4u)$
[3 marks available — 3 marks for the correct factorisation, otherwise 1 mark for the numbers in the brackets correct and 1 mark for either the t's or the u's correct]

7 a) $(2x - 9)(x + 5)$
[2 marks available — 1 mark for correct numbers in brackets, 1 mark for correct signs]

b) $(2x-9)(x+5) = (2x-9)^2$ ***[1 mark]***
$(2x-9)(x+5) - (2x-9)^2 = 0$
$(2x-9)[(x+5)-(2x-9)] = 0$ ***[1 mark]***
$2x-9 = 0$ or $x+5-2x+9 = 0$ ***[1 mark]***
$2x = 9$ or $-x+14 = 0$
$x = 4.5$ or $x = 14$
[1 mark for both solutions]
[4 marks available in total — as above]

8 a) $(5x-6)(x-2)$
[2 marks available — 1 mark for correct numbers in brackets, 1 mark for correct signs]

b) Replacing x with $(x+1)$ in the factorised expression from a)...
$5(x+1)^2 - 16(x+1) + 12 = (5(x+1)-6)((x+1)-2)$ ***[1 mark]***
$= (5x+5-6)(x+1-2)$
$= (5x-1)(x-1)$ ***[1 mark]***
[2 marks available in total — as above]

Page 20: The Quadratic Formula

1 a = 1, b = 7 and c = 2

$$x = \frac{-7 \pm \sqrt{7^2 - 4 \times 1 \times 2}}{2 \times 1} = \frac{-7 \pm \sqrt{41}}{2}$$

$x = -0.30$ or $x = -6.70$
[3 marks available — 1 mark for correct substitution, 1 mark for simplifying and 1 mark for both solutions]

2 a = 1, b = 5 and c = –8

$$x = \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times -8}}{2 \times 1} = \frac{-5 \pm \sqrt{57}}{2}$$

$x = 1.27$ or $x = -6.27$
[3 marks available — 1 mark for correct substitution, 1 mark for simplifying and 1 mark for both solutions]

3 $2a = 6$, so $a = 6 \div 2 = 3$ ***[1 mark]***
$-b = 9$, so $b = -9$ ***[1 mark]***
$4ac = 48$, so $c = 48 \div 4 \div 3 = 4$ ***[1 mark]***
[3 marks available in total — as above]

4 a = 5, b = –6 and c = –3

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 5 \times -3}}{2 \times 5} = \frac{6 \pm \sqrt{96}}{10}$$

$$x = \frac{3 + 2\sqrt{6}}{5} \text{ or } x = \frac{3 - 2\sqrt{6}}{5}$$

[4 marks available — 1 mark for correct substitution, 1 mark for simplifying $\sqrt{96}$ to $4\sqrt{6}$, 1 mark for fully simplifying, 1 mark for both solutions]

Page 21: Completing the Square

1 $-8 \div 2 = -4$, so $a = -4$ and the bit in brackets is $(x-4)^2$.
Expanding the brackets: $(x-4)^2 = x^2 - 8x + 16$.
To complete the square: $6 - 16 = -10$, so $b = -10$.
$x^2 - 8x + 6 = (x-4)^2 - 10$
[3 marks available — 3 marks for the correct answer, otherwise 1 mark for dividing the x-term by 2 to find the value of a, 1 mark for finding the value of b]

2 $(x+3)^2 - 5 = x^2 + 6x + 9 - 5 = x^2 + 6x + 4$
$a = 6$ and $b = 4$
[2 marks available — 1 mark for finding the value of a, 1 mark for finding the value of b]

3 a) Dividing the first two terms by 2: $2(x^2 - 2x) + 11$
$2 \div 2 = 1$, so the first bit is $2[(x-1)^2]$
Expanding the brackets gives: $2(x^2 - 2x + 1) = 2x^2 - 4x + 2$
To complete the square: $11 - 2 = 9$
So $2x^2 - 4x + 11 = 2(x-1)^2 + 9$
[4 marks available — 4 marks for the correct answer, otherwise 1 mark for dividing the first two terms by 2, 1 mark dividing the x-term by 2 to find the value of b, 1 mark for finding the value of c]

b) Minimum value = 9, which occurs at $x = 1$, so the coordinates of the minimum point are (1, 9) ***[1 mark]***

c) This quadratic is u-shaped, so it's always greater than 0 as its minimum value is 9. This means it never crosses the x-axis. ***[1 mark]***

4 Expanding the brackets on the RHS gives the quadratic $mx^2 + 4mx + 4m + p$. Comparing the coefficients of x^2 gives $m = 5$ ***[1 mark]***. Comparing the coefficients of x gives $n = 4m$, so $n = 20$ ***[1 mark]***. Comparing the constant terms gives $14 = 4m + p$, so $p = -6$ ***[1 mark]***.
[3 marks available in total — as above]

Page 22: Algebraic Fractions

1 $$\frac{x^2 - 36}{x^2 + 13x + 42} = \frac{(x+6)(x-6)}{(x+6)(x+7)} = \frac{x-6}{x+7}$$

[3 marks available — 1 mark for correctly factorising the denominator, 1 mark for correctly factorising the numerator, 1 mark for the correct answer]

2 $$\frac{5x^2 + 3x - 14}{25x^2 - 49} = \frac{(5x-7)(x+2)}{(5x-7)(5x+7)} = \frac{x+2}{5x+7}$$

[3 marks available — 1 mark for correctly factorising the denominator, 1 mark for correctly factorising the numerator, 1 mark for the correct answer]

3 $$\frac{x^2 + 5x + 6}{x^2 - 9} \div \frac{4}{3x^2 - 9x} = \frac{x^2 + 5x + 6}{x^2 - 9} \times \frac{3x^2 - 9x}{4}$$
$$= \frac{(x+2)(x+3)}{(x+3)(x-3)} \times \frac{3x(x-3)}{4}$$
$$= \frac{3x(x+2)}{4}$$

[5 marks available — 1 mark for inverting the second fraction and multiplying, 1 mark for each correct factorisation, 1 mark for the correct answer]

4 $$\frac{3x^2 - 4x - 15}{x - 4} \times \frac{3x^3 - 12x^2}{9x^2 - 25} = \frac{(3x+5)(x-3)}{x-4} \times \frac{3x^2(x-4)}{(3x+5)(3x-5)}$$
$$= \frac{3x^2(x-3)}{3x-5}$$

[4 marks available — 1 mark for each correct factorisation, 1 mark for the correct answer]

5 $$\frac{28}{2x+5} + \frac{3}{x} = 4$$
$$\frac{28x + 3(2x+5)}{x(2x+5)} = 4 \quad \textit{[2 marks]}$$
$28x + 6x + 15 = 4x(2x+5)$ ***[1 mark]***
$34x + 15 = 8x^2 + 20x$ ***[1 mark]***
$0 = 8x^2 + 20x - 34x - 15$
$0 = 8x^2 - 14x - 15$ ***[1 mark]***
$(4x+3)(2x-5) = 0$ ***[1 mark]***
$x = -\frac{3}{4}$ or $x = \frac{5}{2}$ ***[1 mark]***
[7 marks available in total — as above]

Page 23: Factorising Cubics

1 a) Add up the coefficients: $1 - 1 - 4 + 4 = 0$ ***[1 mark]***, so $(x-1)$ is a factor ***[1 mark]***.
Use this to find a quadratic factor — start by finding the x^2 term and the number term:
$x^3 - x^2 - 4x + 4 = (x-1)(x^2 \quad - 4)$
Find the x term by comparing coefficients:
$x^3 - x^2 - 4x + 4 = (x-1)(x^2 - 4)$ ***[1 mark for x^2, 1 mark for –4]***
(There isn't actually an x-term in this quadratic.)
Finally, factorise the quadratic factor:
$x^3 - x^2 - 4x + 4 = (x-1)(x-2)(x+2)$
[2 marks — 1 mark for each factor of the quadratic]
[6 marks available in total — as above]
Your working might look a bit different if you started off by putting x = 2 or x = –2 into the equation for y at the start to find a different linear factor. The final answer will be the same though.

b) $x^3 - x^2 - 4x + 4 = 0$
$(x - 1)(x - 2)(x + 2) = 0$
So the solutions are $x = 1$, $x = 2$ and $x = -2$. ***[1 mark]***

2 a) Substitute $x = -2$ into $f(x)$:
$f(-2) = (-2)^3 - 4(-2)^2 - 3(-2) + 18 = -8 - 16 + 6 + 18 = 0$
So, by the Factor Theorem, $(x + 2)$ is a factor of $f(x)$. ***[1 mark]***
Substitute $x = 3$ into $f(x)$:
$f(3) = (3)^3 - 4(3)^2 - 3(3) + 18 = 27 - 36 - 9 + 18 = 0$
So, by the Factor Theorem, $(x - 3)$ is a factor of $f(x)$. ***[1 mark]***
[2 marks available in total — as above]

b) First, fully factorise $x^3 - 4x^2 - 3x + 18$:
You know two linear factors, so compare coefficients to work out the third:
$x^3 - 4x^2 - 3x + 18 = (x + 2)(x - 3)(x - 3)$
(Here, you had to compare the coefficients of the number term — there's +18 on the LHS, so you need to multiply the +2 and the −3 on the RHS by −3 to make +18.)
Now factorise the numerator:
$x^3 - 6x^2 + 9x = x(x^2 - 6x + 9) = x(x - 3)(x - 3)$
Finally, simplify the fraction:
$$\frac{x^3 - 6x^2 + 9x}{x^3 - 4x^2 - 3x + 18} = \frac{x(x-3)(x-3)}{(x+2)(x-3)(x-3)} = \frac{x}{x+2}$$
[3 marks available — 1 mark for fully factorising the numerator, 1 mark for fully factorising the denominator, 1 mark for the correct final answer]

3 The Factor Theorem tells you that when you substitute $x = 1$ into the cubic the answer will be 0 (as $(x - 1)$ is a factor).
So $1^3 + a(1^2) + 11(1) + b = 0$ ***[1 mark]***
which simplifies to $a + b = -12$ ***[1 mark]***
Similarly, $(x + 4)$ is a factor so: $(-4)^3 + a(-4)^2 + 11(-4) + b = 0$
which simplifies to $16a + b = 108$ ***[1 mark]***
Solve the two equations simultaneously:
$16a + b = 108$
$a + b = -12$
Subtracting gives: $15a = 120$, so $a = 8$ ***[1 mark]***
Substituting $a = 8$ into $a + b = -12$ gives $b = -20$ ***[1 mark]***
[5 marks available in total — as above]
You can do this one by writing the cubic as (x − 1)(x + 4)(x + c) then expanding the brackets and comparing coefficients to find the value of c, which you then use to work out a and b.

4 a) Substitute $x = 3$ into $f(x)$:
$f(3) = 3^3 - 7(3) - 6 = 27 - 21 - 6 = 0$
So, by the Factor Theorem, $(x - 3)$ is a factor. ***[1 mark]***

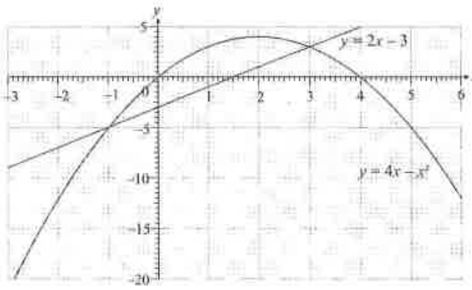
b) Use the linear factor from part a) to find a quadratic factor — start by finding the x^3 term and the number term:
$x^3 - 7x - 6 = (x - 3)(x^2 \quad + 2)$
Find the x term by comparing coefficients:
$x^3 - 7x - 6 = (x - 3)(x^2 + 3x + 2)$ ***[2 marks for correct quadratic factor, otherwise 1 mark for one term correct]***
Factorise the quadratic factor:
$x^3 - 7x - 6 = (x - 3)(x + 2)(x + 1)$ ***[1 mark]***
So the solutions of $f(x) = 0$ are $x = 3$, $x = -2$ and $x = -1$ ***[1 mark]***.
[4 marks available in total — as above]
To find the solutions, you could try substituting values into f(x). You'll find that f(−2) = 0 and f(−1) = 0 (and you know that f(3) = 0), so these are the solutions.

Page 24: Simultaneous Equations and Graphs

1 $x = 3$ and $y = 5$ ***[1 mark]***
These are the x and y coordinates of the point where the two lines cross.

2 a) Point P is the point where $y = 18 - 3x$ crosses the x-axis, so $y = 0$
$0 = 18 - 3x$, so $3x = 18 \Rightarrow x = 6$.
The coordinates of point P are $(6, 0)$. ***[1 mark]***

b) Point Q is the point where the two lines intersect, so solve the simultaneous equations $y = 18 - 3x$ and $y = 2x - 2$:
$3x + y = 18$ and $2x - y = 2$
Adding the equations gives: $5x = 20$, so $x = 4$.
Substituting this value into $y = 2x - 2$ gives $y = 6$.
So the coordinates of point Q are $(4, 6)$.
[3 marks available — 1 mark for some correct working, 1 mark for the correct x-value and 1 mark for the correct y-value]
You could have done this one by setting the two equations equal to each other and solving for x, then using your x-value to find y.

3


$x = -1, y = -5$ and $x = 3, y = 3$
[3 marks available — 1 mark for correctly drawing the line y = 2x − 3, 1 mark for each correct solution]

Page 25: Simultaneous Equations

1 $3x + 5y = 49 \xrightarrow{\times 5} 15x + 25y = 245$ ***[1 mark]***
$5x + 2y = 31 \xrightarrow{\times 3} 15x + 6y = 93$ ***[1 mark]***

$15x + 25y = 245$
$15x + 6y = 93$ −
$19y = 152$
$y = 8$ ***[1 mark]***

$3x + 5y = 49$
$3x = 49 - (5 \times 8)$
$3x = 9$
$x = 3$ ***[1 mark]***

[4 marks available in total — as above]
Your working might look a bit different — for example, you might have multiplied the first equation by 2 and the second equation by 5 to eliminate the y-terms. There's usually more than one way to solve simultaneous equations.

2 Rearrange both equations into the form $ax + by = c$:
$\frac{3y - 4}{x - 2} = 5 \Rightarrow 3y - 4 = 5x - 10 \Rightarrow 5x - 3y = 6$
$3x = 2y + 5 \Rightarrow 3x - 2y = 5$ ***[1 mark for rearranging both equations]***
$5x - 3y = 6 \xrightarrow{\times 3} 15x - 9y = 18$ ***[1 mark]***
$3x - 2y = 5 \xrightarrow{\times 5} 15x - 10y = 25$ ***[1 mark]***

$15x - 9y = 18$
$15x - 10y = 25$ −
$y = -7$ ***[1 mark]***

$3x = 2y + 5$
$3x = (2 \times -7) + 5$
$3x = -9$
$x = -3$ ***[1 mark]***

[5 marks available in total — as above]

3 $2x + y = -6$, so $y = -2x - 6$
$x^2 + (-2x - 6) = 2$ ***[1 mark]***
$x^2 - 2x - 8 = 0$ ***[1 mark]***
$(x + 2)(x - 4) = 0$ ***[1 mark]***
$x = -2$ or $x = 4$
When $x = -2$, $y = (-2 \times -2) - 6 = -2$
When $x = 4$, $y = (-2 \times 4) - 6 = -14$
So the solutions are $x = -2, y = -2$ ***[1 mark]***
and $x = 4, y = -14$ ***[1 mark]***
[5 marks available in total — as above]

4 $y = x + 3$ so $x = y - 3$
$3(y - 3) + 2y^2 = 11$ ***[1 mark]***
$3y - 9 + 2y^2 = 11$
$2y^2 + 3y - 20 = 0$ ***[1 mark]***
$(2y - 5)(y + 4) = 0$ ***[1 mark]***
So $y = 2.5$ or $y = -4$
When $y = 2.5$, $x = 2.5 - 3 = -0.5$
When $y = -4$, $x = -4 - 3 = -7$
So the solutions are $x = -0.5$, $y = 2.5$ ***[1 mark]***
and $x = -7$, $y = -4$ ***[1 mark]***
[5 marks available in total — as above]
You could have substituted $y = x + 3$ into $3x + 2y^2 = 11$ instead — you'd have ended up with the same solutions.

5 $y = x + 4$, so $3x^2 - (x + 4)^2 = -22$ ***[1 mark]***
$3x^2 - x^2 - 8x - 16 = -22$
$2x^2 - 8x + 6 = 0$ ***[1 mark]***
$x^2 - 4x + 3 = 0$
$(x - 3)(x - 1) = 0$ ***[1 mark]***, so $x = 3$ or $x = 1$
When $x = 3$, $y = 3 + 4 = 7$
When $x = 1$, $y = 1 + 4 = 5$
So the solutions are $x = 3$, $y = 7$ ***[1 mark]*** and $x = 1$, $y = 5$ ***[1 mark]***
[5 marks available in total — as above]

Pages 26-27: Inequalities

1 $12 < 5p \leq 30$, so $2.4 < p \leq 6$
$p = 3, 4, 5$ or 6
[3 marks available — 3 marks for all four values, otherwise 2 marks for three correct values, otherwise 1 mark for two correct values]

2 a) $6q - 8 < 40$, so $6q < 48$ ***[1 mark]*** and $q < 8$ ***[1 mark]***
[2 marks available in total — as above]

b) $\frac{3x}{4} \leq 9$, so $3x \leq 36$ ***[1 mark]*** and $x \leq 12$ ***[1 mark]***
[2 marks available in total — as above]

3 a) $7x - 2 < 2x - 42$, so $5x < -40$ ***[1 mark]*** and $x < -8$ ***[1 mark]***
[2 marks available in total — as above]

b) $9 - 4x > 17 - 2x$, so $-8 > 2x$ ***[1 mark]*** and $x < -4$ ***[1 mark]***
[2 marks available in total — as above]

4 a) $3 \leq 2p + 5 \leq 15$, so $-2 \leq 2p \leq 10$ ***[1 mark]***, so $-1 \leq p \leq 5$ ***[1 mark]***
[2 marks available in total — as above]

b) $q^2 - 9 > 0$, so $q^2 > 9$. The solutions to $q^2 = 9$ are $q = 3$ and $q = -3$, so $q^2 > 9$ when $q < -3$ and $q > 3$
[3 marks available — 1 mark for finding the correct solutions to $q^2 = 9$, 1 mark for $q < -3$, 1 mark for $q > 3$]
If you need to, sketch the graph of $y = q^2 - 9$ to see where it's greater than 0 (i.e. above the x-axis).

5 The solutions of $w^2 = 25$ are $w = 5$ and $w = -5$
So $w^2 \geq 25$ when $w \geq 5$ ***[1 mark]*** or when $w \leq -5$ ***[1 mark]***
[2 marks available in total — as above]

6 a) $3x^2 - 5x - 2 = 0$ factorises to give $(3x + 1)(x - 2) = 0$.
The graph of $y = 3x^2 - 5x - 2$ is a u-shaped quadratic that crosses the x-axis at $x = -\frac{1}{3}$ and $x = 2$, and the graph is below (or equal to) 0 between these points. So $-\frac{1}{3} \leq x \leq 2$.
[3 marks available — 1 mark for factorising the quadratic to find the solutions, 1 mark for $-\frac{1}{3} \leq x$, 1 mark for $x \leq 2$]

b) $2x^2 > x + 1$, so $2x^2 - x - 1 > 0$
$2x^2 - x - 1 = 0$ factorises to give $(2x + 1)(x - 1) = 0$.
The graph of $y = 2x^2 - x - 1$ is a u-shaped quadratic that crosses the x-axis at $x = -\frac{1}{2}$ and $x = 1$, and the graph is positive (i.e. > 0) when x is less than $-\frac{1}{2}$ or greater than 1.
So $x < -\frac{1}{2}$ and $x > 1$.
[3 marks available — 1 mark for factorising the quadratic to find the solutions, 1 mark for $x < -\frac{1}{2}$, 1 mark for $x > 1$]

7 $(x + 1)^2$ must be positive or 0.
The smallest value occurs when $x = -1$ as this makes $(x + 1)^2$ equal to 0. The greatest value occurs when $x = -8$ as this makes $(x + 1)^2$ equal to 49. So $0 \leq (x + 1)^2 \leq 49$.
[3 marks available — 1 mark for a suitable method, 1 mark for 0 and 1 mark for 49]
You might find it helpful to sketch the graph of $y = (x + 1)^2$ to see what's going on.

8 a) The volume of cuboid A is $5x(x + 1) = 5x^2 + 5x$. ***[1 mark]***
The volume of cuboid B is $6x^2$. ***[1 mark]***
$5x^2 + 5x > 6x^2$ so $x^2 - 5x < 0$. ***[1 mark]***
[3 marks available in total — as above]

b) $x^2 - 5x = 0$ factorises to give $x(x - 5) = 0$ ***[1 mark]***.
The graph of $y = x^2 - 5x$ is a u-shaped quadratic that crosses the x-axis at $x = 0$ and $x = 5$, and the graph is negative (i.e. < 0) between these points. So $0 < x < 5$ ***[1 mark]***.
The largest integer solution is $x = 4$ ***[1 mark]***, so the greatest possible volume of cuboid A is $5(4)(4 + 1) = 5 \times 4 \times 5 = 100\text{ cm}^3$. ***[1 mark]***
[4 marks available in total — as above]

Page 28: Algebraic Proof

1 $(2n + 1)^2 - 5(2n + 1) = (4n^2 + 4n + 1) - (10n + 5)$
$= 4n^2 + 4n + 1 - 10n - 5$
$= 4n^2 - 6n - 4$ ***[1 mark]*** $= 2(2n^2 - 3n - 2)$
$= 2x$ (where $x = 2n^2 - 3n - 2$) ***[1 mark]***
An integer multiplied by 2 is always even, so $2x$ will always be even. Therefore, $(2n + 1)^2 - 5(2n + 1)$ will always be even.
[2 marks available in total — as above]

2 n is an integer. The nth term is $6n - 3$ and the $(n + 1)$th term is $6(n + 1) - 3 = 6n + 3$ ***[1 mark]***. So the sum of the two consecutive terms is $(6n - 3) + (6n + 3) = 12n$ ***[1 mark]***, which is a multiple of 12 for any integer value of n.
[2 marks available in total — as above]

3 n is an integer. $2n + 1$ represents any odd number.
$(2n + 1)^2$ ***[1 mark]*** $= 4n^2 + 4n + 1$ ***[1 mark]*** $= 4(n^2 + n) + 1$
$= 4x + 1$ (where $x = n^2 + n$) ***[1 mark]***
Any integer multiplied by 4 is a multiple of 4, so $4x$ must be a multiple of 4 and therefore the square of any odd number will always be one more than a multiple of 4.
[3 marks available in total — as above]

4 $(2n + 5)^2 - (n + 1)^2 = (4n^2 + 20n + 25) - (n^2 + 2n + 1)$ ***[1 mark]***
$= 4n^2 + 20n + 25 - n^2 - 2n - 1 = 3n^2 + 18n + 24$ ***[1 mark]***
$= 3(n^2 + 6n + 8) = 3x$ (where $x = n^2 + 6n + 8$) ***[1 mark]***
Any integer multiplied by 3 is a multiple of 3, so $3x$ must be a multiple of 3 and therefore $(2n + 5)^2 - (n + 1)^2$ will be a multiple of 3 for any positive integer value of n.
[3 marks available in total — as above]
You could use the difference of two squares to simplify the expression.

5 n is an integer. $2n$ represents any even number, so the product of 3 consecutive even numbers is given by $2n(2n + 2)(2n + 4)$. ***[1 mark]***
$2n(2n + 2)(2n + 4) = (4n^2 + 4n)(2n + 4) = 8n^3 + 24n^2 + 16n$ ***[1 mark]***
$= 8(n^3 + 3n^2 + 2n) = 8x$ (where $x = n^3 + 3n^2 + 2n$) ***[1 mark]***.
Any integer multiplied by 8 is a multiple of 8, so $8x$ must be a multiple of 8 and therefore the product of any 3 consecutive even numbers will be a multiple of 8.
[3 marks available in total — as above]
You might have taken out a factor of 8 a bit earlier:
$2n(2n + 2)(2n + 4) = 2n[2(n + 1)][2(n + 2)] = 8n(n + 1)(n + 2)$

Page 29: Sequences

1 Sequence: 4 6 10 16 24
First difference: 2 4 6 8 *[1 mark]*
Second difference: 2 2 2 *[1 mark]*
The second differences are constant so the sequence is quadratic.
Coefficient of $n^2 = 2 \div 2 = 1$.
Sequence given by n^2: 1 4 9 16 25
Actual sequence – n^2 sequence:
3 2 1 0 –1 *[1 mark]*
Difference: –1 –1 –1 –1 *[1 mark]*
So this is a linear sequence with nth term $-n + 4$.
So the nth term of the original sequence is $n^2 - n + 4$ *[1 mark]*.
[5 marks available in total — as above]

2 Sequence: 13 18 25 34 45
First difference: 5 7 9 11 *[1 mark]*
Second difference: 2 2 2 *[1 mark]*
The second differences are constant so the sequence is quadratic.
Coefficient of $n^2 = 2 \div 2 = 1$.
Sequence given by n^2: 1 4 9 16 25
Actual sequence – n^2 sequence:
12 14 16 18 20 *[1 mark]*
Difference: 2 2 2 2 *[1 mark]*
So this is a linear sequence with nth term $2n + 10$.
So the nth term of the original sequence is $n^2 + 2n + 10$ *[1 mark]*.
[5 marks available in total — as above]

3 a) The numerators form the linear sequence 3, 5, 7, 9...
which has nth term $2n + 1$ *[1 mark]*.
The denominators form the linear sequence 4, 7, 10, 13...
which has nth term $3n + 1$ *[1 mark]*.
So the nth term of the sequence is $\frac{2n+1}{3n+1}$ *[1 mark]*.
[3 marks available in total — as above]

b) $\frac{2n+1}{3n+1} = \frac{\frac{2n}{n}+\frac{1}{n}}{\frac{3n}{n}+\frac{1}{n}} = \frac{2+\frac{1}{n}}{3+\frac{1}{n}}$ *[1 mark]*

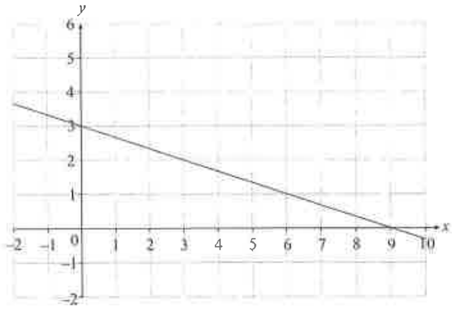
As $n \to \infty$, $\frac{1}{n} \to 0$, so $\frac{2n+1}{3n+1} \to \frac{2}{3}$ *[1 mark]*
[2 marks available in total — as above]

4 Substitute $n = 1$ into the expression for n: $\frac{a+1}{b+3} = 1$,
so $a + 1 = b + 3$, or $a = b + 2$ *[1 mark]*.
$\frac{an+1}{bn+3} \to \frac{a}{b}$ as $n \to \infty$, so $\frac{a}{b} = 2$, or $a = 2b$ *[1 mark]*.
Substitute $a = b + 2$ into $a = 2b$: $b + 2 = 2b$, so $b = 2$ *[1 mark]*
and $a = 4$ *[1 mark]*.
The nth term formula is $\frac{4n+1}{2n+3}$, so when $n = 6$,
the term is $\frac{4(6)+1}{2(6)+3} = \frac{25}{15} = \frac{5}{3}$ *[1 mark]*.
[5 marks available in total — as above]

Section Three — Graphs, Functions and Calculus

Pages 30-32: Straight-Line Graphs

1



[2 marks available — 1 mark for a line with gradient $-\frac{1}{3}$ and 1 mark for a y-intercept of 3]
To draw this graph, you could either create a table of values and plot the points, or you could set y = 0 and x = 0 and join up the points.

2 Using $y = mx + c$, where m is the gradient, and c is the y-intercept:
Find the gradient, m:
Using the points (0.1, 0.2) and (–0.3, 0),
$m = \frac{(0.2-0)}{(0.1-(-0.3))} = 0.5$ *[1 mark]*
Find c:
When $x = 0$, $y = 0.15$, so c = 0.15
So, $y = 0.5x + 0.15$ *[1 mark]*
[2 marks available in total — as above]

3 a) Rearrange $3x + 4y = 12$ to give $y = 3 - \frac{3}{4}x$
Find the gradient (m) by comparing to $y = mx + c$:
$m = -\frac{3}{4}$ *[1 mark]*

b) When $y = 0$, $x = 4$ so Q is (4, 0) *[1 mark]*

c) At $x = 0$, $4y = 12$, so $y = 3$. So P is the point (0, 3)
which means the y-intercept of Line B is 3.
The gradient of Line B is $\frac{(3-0)}{(0-(-1))} = 3$ *[1 mark]*
So, the equation of line B is $y = 3x + 3$ *[1 mark]*
[2 marks available in total — as above]

4 $m = \frac{-7-17}{5-(-1)} = -4$ *[1 mark]*
Equation of line is given by $y - y_1 = m(x - x_1)$, so:
$y - 17 = -4(x - (-1))$
So $y = -4x + 13$ *[1 mark]*
[2 marks available in total — as above]

5 a) $m = \frac{11-2}{5-2} = 3$ *[1 mark]*
Equation of line is given by $y - y_1 = m(x - x_1)$, so:
$y - 2 = 3(x - 2)$
$y = 3x - 4$ *[1 mark]*
When $x = 0$, $y = -4$ so point A is (0, –4) *[1 mark]*
When $y = 0$, $x = 4 \div 3$, so point B is $\left(\frac{4}{3}, 0\right)$ *[1 mark]*
[4 marks available in total — as above]

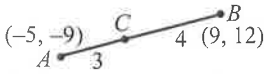
b) Using gradient from part a), m = 3 *[1 mark]*
Equation of line is given by $y - y_1 = m(x - x_1)$, so:
$y - (-8) = 3(x - 4)$
So $y = 3x - 20$ *[1 mark]*
[2 marks available in total — as above]

6 $5x + 3 = 3x + 7$ *[1 mark]*, so $x = 2$ *[1 mark]*
so $y = 13$ and point M is (2, 13) *[1 mark]*
Gradient of perpendicular line $= \frac{-1}{4} = -0.25$ *[1 mark]*
Equation of line is given by $y - y_1 = m(x - x_1)$, so:
$y - 13 = -0.25(x - 2)$
$y = -0.25x + 13.5$ *[1 mark]*
[5 marks available in total — as above]

Page 33: Coordinates and Ratio

1 a) Substitute $(1, k)$ into $5y - x = 9$:
$5k - 1 = 9 \Rightarrow 5k = 10 \Rightarrow k = 2$. ***[1 mark]***

b) Mid-Point $= \left(\frac{1+6}{2}, \frac{2+3}{2}\right) = (3.5, 2.5)$.
[2 marks available — 1 mark for x-coordinate, 1 mark for y-coordinate]

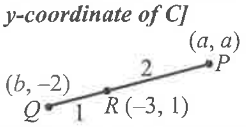
2


Difference in x-coordinates from A to $B = 9 - (-5) = 14$
Difference in y-coordinates from A to $B = 12 - (-9) = 21$

C is $\frac{3}{7}$ of the way from A to B, so:

x: $\frac{3}{7} \times 14 = 6$ and y: $\frac{3}{7} \times 21 = 9$
x-coordinate: $-5 + 6 = 1$ and y-coordinate: $-9 + 9 = 0$
So C is point $(1, 0)$

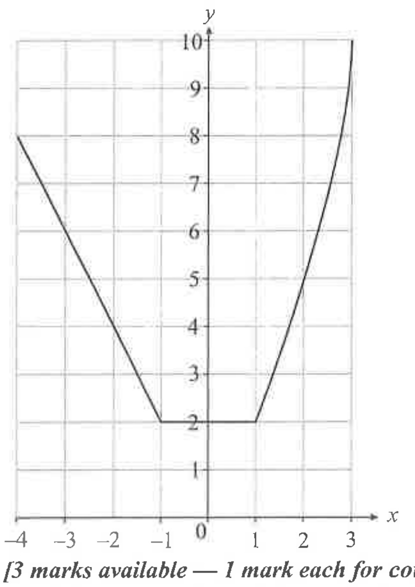
[4 marks available — 1 mark for finding the difference in x- and y-coordinates from A to B, 1 mark for multiplying these by $\frac{3}{7}$, 1 mark for the correct x-coordinate of C and 1 mark for the correct y-coordinate of C]

3


Difference in y-coordinates from Q to $R = 1 - (-2) = 3$ ***[1 mark]***
So, using the ratio, the difference in y-coordinates from R to P is $2 \times 3 = 6$ ***[1 mark]***. So $a = 1 + 6 = 7$ ***[1 mark]***.
Difference in x-coordinates from R to $P = a - (-3) = 7 - (-3) = 10$
So, using the ratio, the difference in x-coordinates from Q to R is $\frac{1}{2} \times 10 = 5$ ***[1 mark]***. So $b + 5 = -3$, so $b = -8$ ***[1 mark]***.
[5 marks available in total — as above]

Pages 34-35: Functions

1 a) $f(-3.5) = -2 \times -3.5 = 7$ ***[1 mark]***

b)


[3 marks available — 1 mark each for correctly plotting the graph for $x < -1$, $-1 \leq x \leq 1$ and $x > 1$]

c) Either $-2x = 26$, i.e. $x = -13$ or $x^2 + 1 = 26$. i.e. $x^2 = 25$
so $x = 5$ ($x = -5$ is outside the range for this function)
[3 marks available — 1 mark for $x = -13$, 1 mark for solving $x^2 + 1 = 26$, 1 mark for $x = 5$ (lose 1 mark if $x = -5$ is included)]

2 You can't take the square root of a negative number, so find the values of x which make $2x + 7 < 0$ ***[1 mark]***
$2x + 7 < 0$
$2x < -7$
$x < -3.5$ ***[1 mark]***
[2 marks available in total — as above]

3 a) $g(3) = 3 - 3^2 = -6$ ***[1 mark]***

b) $g(x) \leq 3$ ***[1 mark]***

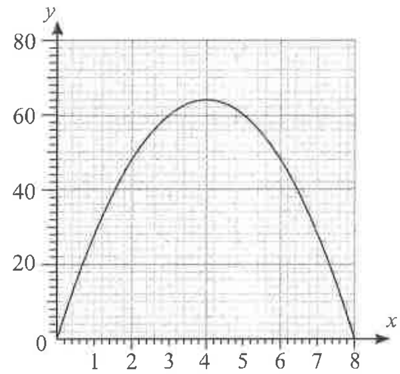
4 Substitute the bounds of the domain and range into the function:
$-6n - m = -27$ (call this equation 1)
$3n - m = 9$ (call this equation 2)
Subtract equation 1 from equation 2:
$9n = 36$, so $n = 4$
Substitute $n = 4$ into equation 2:
$12 - m = 9$, so $m = 3$
[4 marks available — 1 mark for substituting values into the function to find two simultaneous equations, 1 mark for subtracting one equation from the other, 1 mark for finding the value of n and 1 mark for finding the value of m]
Here, f(x) is a decreasing function, so the lower limit of the domain will give the upper limit of the range, and vice versa.

5 a) $g(-2) = 8 - 3(-2) = 14$
$g(4) = 8 - 3(4) = -4$
So $-4 < g(x) \leq 14$
[2 marks available — 1 mark for correct values, 1 mark for correct inequality symbols]

b) $g(2 - x) = 8 - 3(2 - x)$ ***[1 mark]***
$= 8 - 6 + 3x$
$= 3x + 2$ ***[1 mark]***
[2 marks available in total — as above]

c) If $g(2 - x) = -1$, then from part b), $3x + 2 = -1$
So $x = -1$ ***[1 mark]***

Page 36: Quadratic Graphs

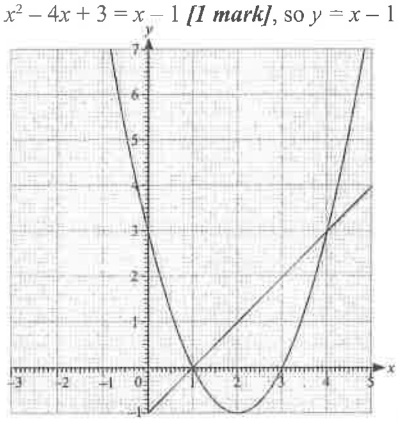
1 a)


[2 marks available — 1 mark for plotting correct values and 1 mark for a smooth curve through these points]

b) 64 ***[1 mark]***

2 a) $x = 1$ and $x = 3$ ***[1 mark]***

b) $x^2 - 5x + 4 = 0$
$x^2 - 4x + 4 = x$
$x^2 - 4x + 3 = x - 1$ ***[1 mark]***, so $y = x - 1$ ***[1 mark]***

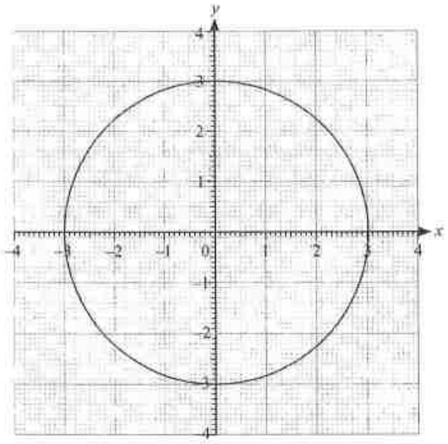


[1 mark]

So $x = 1$ and $x = 4$ ***[1 mark]***
[4 marks available in total — as above]

Pages 37-38: Equation of a Circle

1



[2 marks available — 1 mark for a circle of radius 3, 1 mark for the centre point of the circle at (0, 0)]

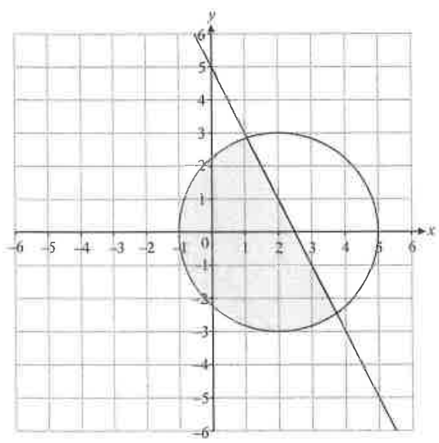
2 Rearrange the equation and complete the square:
$x^2 - 2x + y^2 - 10y + 21 = 0$
$(x - 1)^2 - 1 + (y - 5)^2 - 25 + 21 = 0$
[1 mark for each completed square]
$(x - 1)^2 + (y - 5)^2 = 5$ *[1 mark]*
Compare with $(x - a)^2 + (y - b)^2 = r^2$:
centre = (1, 5) *[1 mark]*, radius = $\sqrt{5}$ *[1 mark]*
[5 marks available in total — as above]

3 The radius of the circle is 10 – 7 = 3
So the equation of the circle is $(x - 2)^2 + (y - 7)^2 = 9$
[3 marks available — 1 mark for finding the radius, 1 mark for either $(x - 2)^2$ or $(y - 7)^2$ and 1 mark for a correct full equation]
There's no need to expand the brackets in the equation, but if you did, the correct equation is $x^2 - 4x + y^2 - 14y + 44 = 0$.

4 a) x-coordinate of centre = 5
y-coordinate of centre = (14 + 6) ÷ 2 = 10
Equation of circle: $(x - 5)^2 + (y - 10)^2 = 25$
[4 marks available — 1 mark each for finding the x- and y-coordinate of the centre, 1 mark for $(x - 5)^2 + (y - 10)^2$ and 1 mark for 25 (or 5^2)]

b) Substitute $x = 8$ and $y = 14$ into the LHS of the circle equation:
$(8 - 5)^2 + (14 - 10)^2 = 3^2 + 4^2 = 25$ *[1 mark]*
This equals the RHS of the equation, so (8, 14) lies on the circle.

5 a)



[3 marks available — 1 mark for a correctly drawn straight line, 1 mark for drawing a circle centred on (2, 0), 1 mark for drawing a circle with radius 3]

b) See graph above *[1 mark]*

Page 39: Differentiation

1 $\frac{dy}{dx} = 6x^2 + 2x - 8$ *[1 mark]*

2 $\frac{dy}{dx} = -2x$ *[1 mark]*
At $x = -1$: $\frac{dy}{dx} = -2(-1) = 2$ *[1 mark]*
At $x = 2$: $\frac{dy}{dx} = -2(2) = -4$ *[1 mark]*
[3 marks available in total — as above]

3 a) $\frac{dy}{dx} = 12x^2 - 2$ *[1 mark]*

b) The gradient is 1 when $\frac{dy}{dx} = 1$, i.e. when $12x^2 - 2 = 1$ *[1 mark]*
$12x^2 - 2 = 1$
$x^2 = \frac{1}{4}$
So $x = \frac{1}{2}$ *[1 mark]* or $x = -\frac{1}{2}$ *[1 mark]*
[3 marks available in total — as above]

4 Multiply out the numerator: $5x^3(x^2 + 2) = 5x^5 + 10x^3$
Simplify the denominator: $(\sqrt{x})^2 = x$
Rewrite and simplify the fraction:
$y = \frac{5x^5 + 10x^3}{x} = 5x^4 + 10x^2$
$\frac{dy}{dx} = 20x^3 + 20x$
[5 marks available — 1 mark for multiplying out the numerator, 1 mark for simplifying the denominator, 1 mark for simplifying the fraction and 1 mark for each correct term in the final expression]

5 $(2x + 1)^2 = 4x^2 + 4x + 1$ *[1 mark]*
So $y = x(2x + 1)^2 = 4x^3 + 4x^2 + x$ *[1 mark]*
$\frac{dy}{dx} = 12x^2 + 8x + 1$ *[1 mark for $12x^2$, 1 mark for $8x + 1$]*
When $x = -1$, $\frac{dy}{dx} = 12(-1)^2 + 8(-1) + 1$ *[1 mark]* = 5 *[1 mark]*
[6 marks available in total — as above]

Page 40: Finding Tangents and Normals

1 a) $\frac{dy}{dx} = 6x^2 - 8x - 4$
[2 marks available — 2 marks for all 3 terms correct, otherwise 1 mark for 2 terms correct]

b) To find the gradient, put $x = 2$ into the answer to part a):
$6(2^2) - 8(2) - 4 = 4$ *[1 mark]*.

c) The gradient of the normal is $-1 \div 4 = -0.25$ *[1 mark]*.
At $x = 2$, the y-value is $2(2^3) - 4(2^2) - 4(2) + 12 = 4$ *[1 mark]*.
Putting these values into the formula $y - y_1 = m(x - x_1)$ gives:
$y - 4 = -0.25(x - 2)$ *[1 mark]*
$y = -0.25x + 4.5$
[3 marks available in total — as above]
You could also give your answer in the form $x + 4y = 18$ by multiplying through by 4 to get rid of the decimals.

2 $y = x(x - 2)(x + 2) = x^3 - 4x$ *[1 mark]*
$\frac{dy}{dx} = 3x^2 - 4$ *[1 mark]*
When $x = 1$, $\frac{dy}{dx} = 3(1^2) - 4 = -1$ *[1 mark]*, so the gradient of the tangent (m) is –1. When $x = 1$, $y = 1^3 - 4(1) = -3$ *[1 mark]*
Putting these values into the formula $y - y_1 = m(x - x_1)$ gives:
$y - (-3) = -1(x - 1)$ *[1 mark]*
So $y = -x - 2$
[5 marks available in total — as above]

3 $\frac{dy}{dx} = 4 - 2x$ *[1 mark]*
When $x = 3$, $\frac{dy}{dx} = 4 - 2(3) = -2$ *[1 mark]*
The equation of the tangent is given by:
$(y - 7) = -2(x - 3)$
$y = -2x + 13$ *[1 mark]*
When $x = 0$, $y = 13$ *[1 mark]*, so A is the point (0, 13)
When $y = 0$, $x = 6.5$ *[1 mark]*, so B is the point (6.5, 0)
Area of $OAB = 0.5 \times 6.5 \times 13 = 42.25$ *[1 mark]*
[6 marks available in total — as above]

Pages 41-42: Stationary Points

1 $y = (x + 4)(x - 8) = x^2 - 4x - 32$
$\frac{dy}{dx} = 2x - 4$ *[1 mark]*
$\frac{dy}{dx} = 0$ gives $x = 2$ *[1 mark]*
$y = (2)^2 - 4(2) - 32$
$= -36$ *[1 mark]*
So coordinates of the stationary point on the curve are (2, –36).
[3 marks available in total — as above]

2 a) $y = 6 + \frac{2x^3 - 6x^2 + 6x}{3} = 6 + \frac{2}{3}x^3 - 2x^2 + 2x$
$\frac{dy}{dx} = 2x^2 - 4x + 2$
[2 marks available — 2 marks for fully correct answer, otherwise 1 mark for at least one term correct]

b) Stationary points occur when $2x^2 - 4x + 2 = 0$, so $x^2 - 2x + 1 = 0$
i.e. when $(x - 1)^2 = 0$ *[1 mark]*.
So stationary point occurs when $x = 1$ *[1 mark]*.
When $x = 1$:
$y = 6 + \frac{2(1)^3 - 6(1)^2 + 6(1)}{3} = 6\frac{2}{3}$ *[1 mark]*
So coordinates of the stationary point on the curve are $\left(1, 6\frac{2}{3}\right)$.
[3 marks available in total — as above]

c) $\frac{d^2y}{dx^2} = 4x - 4$ *[1 mark]*. At the stationary point, $\frac{d^2y}{dx^2} = 4 - 4 = 0$,
so find the gradient either side of the stationary point.
At $x = 0$, $\frac{dy}{dx} = 2 > 0$ and at $x = 2$, $\frac{dy}{dx} = 2 > 0$,
so the point is a point of inflection *[1 mark]*.
[2 marks available in total — as above]

3 a) First, multiply out the function to get $y = x^3 - 2x^2 + x$
$\frac{dy}{dx} = 3x^2 - 4x + 1$ ***[2 marks for all three terms correct, otherwise 1 mark for two terms correct]***
$\frac{dy}{dx} = 0$ means that $3x^2 - 4x + 1 = (3x - 1)(x - 1) = 0$
So the stationary points occur at $x = 1$ *[1 mark]*
and $x = \frac{1}{3}$ *[1 mark]* which gives:
When $x = 1$, $y = 1(1 - 1)^2 = 0$ *[1 mark]*
When $x = \frac{1}{3}$, $y = \frac{1}{3}\left(\frac{1}{3} - 1\right)^2 = \frac{4}{27}$ *[1 mark]*
So the stationary points have coordinates:
$(1, 0)$ and $\left(\frac{1}{3}, \frac{4}{27}\right)$.
[6 marks available in total — as above]

b) $\frac{d^2y}{dx^2} = 6x - 4$ *[1 mark]*
At $x = 1$, $\frac{d^2y}{dx^2} = 2$ (> 0), so (1, 0) is a minimum *[1 mark]*
At $x = \frac{1}{3}$, $\frac{d^2y}{dx^2} = -2$ (< 0), so $\left(\frac{1}{3}, \frac{4}{27}\right)$ is a maximum *[1 mark]*
[3 marks available in total — as above]
Instead of differentiating again, you could just find the gradient either side of the stationary point and compare the signs.

c) At $x = 2$, $\frac{dy}{dx} = 3(2)^2 - 4(2) + 1 = 12 - 8 + 1 = 5$,
which is positive, so the function is increasing. *[1 mark]*

4 a) $y = x^3 - 6x^2 + 12x + 5$ so
$\frac{dy}{dx} = 3x^2 - 12x + 12$ ***[2 marks for all three terms correct, otherwise 1 mark for two terms correct]***
$\frac{dy}{dx} = 0$ means that $3x^2 - 12x + 12 = 0$, so $x^2 - 4x + 4 = 0$,
which factorises to give $(x - 2)^2 = 0$ *[1 mark]*
So there is just one stationary point at $x = 2$ *[1 mark]*
[4 marks available in total — as above]

b) Find the gradient of the curve at points either side of $x = 2$:
When $x = 1.5$, $\frac{dy}{dx} = 3(1.5)^2 - 12(1.5) + 12 = 0.75$ (> 0)
When $x = 2.5$, $\frac{dy}{dx} = 3(2.5)^2 - 12(2.5) + 12 = 0.75$ (> 0)
So the stationary point is a point of inflection.
[2 marks available — 1 mark for finding the gradient at two points either side of x = 2 and 1 mark for identifying point of inflection]

5 a) $y = -x^3 + 3x^2 + ax + b$ so
$\frac{dy}{dx} = -3x^2 + 6x + a$ ***[2 marks for all three terms correct, otherwise 1 mark for two terms correct]***
$\frac{dy}{dx} = 0$ at point A, so $-3(-1)^2 + 6(-1) + a = 0$ *[1 mark]*
So $a = 9$ *[1 mark]*
The y-coordinate of point A is 5, so putting $x = -1$ and $y = 5$ into f(x) gives:
$5 = -(-1)^3 + 3(-1)^2 + 9(-1) + b$ which gives $b = 10$ *[1 mark]*
[5 marks available in total — as above]

b) $\frac{dy}{dx} = 0$ means that $-3x^2 + 6x + 9 = 0$, so $-x^2 + 2x + 3 = 0$
So $-(x + 1)(x - 3) = 0$ *[1 mark]*
$x = -1$ or $x = 3$
The x-coordinate of point B must be 3 *[1 mark]*
[2 marks available in total — as above]

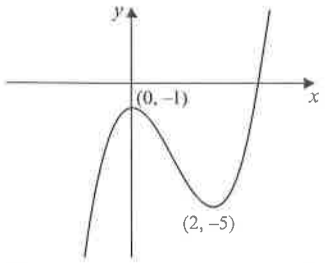
c) The function f(x) is decreasing when $\frac{dy}{dx} < 0$ for the graph $y = $ f(x).
From the sketch, there are two intervals for which $\frac{dy}{dx} < 0$:
$x < -1$ and $x > 3$
[2 marks available — 1 mark for each correct interval]

Page 43: Curve Sketching

1 a) $\frac{dy}{dx} = 3x^2 - 6x$ *[1 mark]*
$\frac{dy}{dx} = 0$ means that $3x^2 - 6x = 0$, so $x^2 - 2x = 0$,
which factorises to give $x(x - 2) = 0$
So the stationary points occur at $x = 0$ *[1 mark]*
and $x = 2$ *[1 mark]*, which gives:
$y = $ f(0) $= 0 - 0 - 1 = -1$ *[1 mark]* and
$y = $ f(2) $= 8 - 12 - 1 = -5$ *[1 mark]*
So the coordinates are (0, –1) and (2, –5).
[5 marks available in total — as above]

b) $\frac{d^2y}{dx^2} = 6x - 6$ *[1 mark]*
At $x = 0$, $\frac{d^2y}{dx^2} = -6$ (< 0), so (0, –1) is a maximum *[1 mark]*.
at $x = 2$, $\frac{d^2y}{dx^2} = 6$ (> 0), so (2, –5) is a minimum *[1 mark]*.
[3 marks available in total — as above]

c)



[3 marks available — 1 mark for a curve with one minimum and one maximum, 1 mark for maximum on y-axis and correctly labelled, 1 mark for minimum in lower-right quadrant and correctly labelled]

2 a) Expand the brackets: $y = -x^2 - 2x + ax + 2a$

$\frac{dy}{dx} = -2x - 2 + a$ *[1 mark]*

$\frac{dy}{dx} = 0$ when $x = 2$, so $-2(2) - 2 + a = 0$ [1 mark]

Which gives $a = 6$ *[1 mark]*

[3 marks available in total — as above]

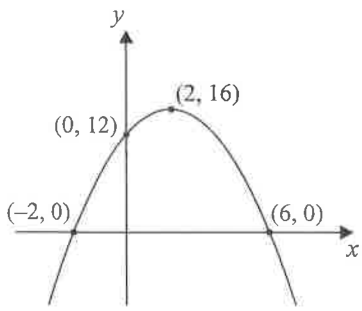
b) $\frac{d^2y}{dx^2} = -2$ (< 0), so the stationary point is a maximum. *[1 mark]*

$f(2) = (2 + 2)(6 - 2) = 16$ *[1 mark]*.

This means that the maximum value of $f(x)$ is 16, so $f(x) = 20$ has no real solutions.

[2 marks available in total — as above]

c)



[3 marks available — 1 mark for a curve with the correct shape, 1 mark for x-intercepts correctly labelled and 1 mark for y-intercept and stationary point correctly labelled]

Section Four — Matrices and Geometry

Page 44: Matrices

1 a) $3\begin{pmatrix}2 & 5\\1 & 7\end{pmatrix} = \begin{pmatrix}6 & 15\\3 & 21\end{pmatrix}$ *[1 mark]*

b) $-5\begin{pmatrix}-2 & 5\\11 & 3\end{pmatrix} = \begin{pmatrix}10 & -25\\-55 & -15\end{pmatrix}$ *[1 mark]*

c) $\begin{pmatrix}4 & 3\\7 & 2\end{pmatrix}\begin{pmatrix}1 & 5\\3 & 9\end{pmatrix} = \begin{pmatrix}(4\times1)+(3\times3) & (4\times5)+(3\times9)\\(7\times1)+(2\times3) & (7\times5)+(2\times9)\end{pmatrix}$

$= \begin{pmatrix}13 & 47\\13 & 53\end{pmatrix}$

[2 marks available — 2 marks for all 4 entries correct, otherwise 1 mark for 2 or 3 entries correct]

d) $\begin{pmatrix}7 & -1\\12 & 2\end{pmatrix}\begin{pmatrix}3 & -2\\2 & 5\end{pmatrix} = \begin{pmatrix}(7\times3)+(-1\times2) & (7\times-2)+(-1\times5)\\(12\times3)+(2\times2) & (12\times-2)+(2\times5)\end{pmatrix}$

$= \begin{pmatrix}19 & -19\\40 & -14\end{pmatrix}$

[2 marks available — 2 marks for all 4 entries correct, otherwise 1 mark for 2 or 3 entries correct]

2 $\mathbf{MN} = \begin{pmatrix}3 & -2\\1 & 4\end{pmatrix}\begin{pmatrix}5\\-1\end{pmatrix} = \begin{pmatrix}(3\times5)+(-2\times-1)\\(1\times5)+(4\times-1)\end{pmatrix}$

$= \begin{pmatrix}17\\1\end{pmatrix}$

[2 marks available — 1 mark for 17, 1 mark for 1]

3 $\begin{pmatrix}0 & x\\-\frac{1}{2} & 1\end{pmatrix}\begin{pmatrix}\frac{2}{3} & -2\\y & 0\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

$\begin{pmatrix}xy & 0\\y-\frac{1}{3} & 1\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

So $xy = 1$ and $y - \frac{1}{3} = 0$, which gives $y = \frac{1}{3}$ and $x = 3$.

[4 marks available — 2 marks for all four entries correct and matrix set = to identity matrix, otherwise 1 mark for 2 or 3 entries correct, 1 mark for the correct value of x, 1 mark for the correct value of y]

Page 45: Matrix Transformations

1 From the top row of the matrix multiplication, $3a - 10 = 2$, so $3a = 12$ which gives $a = 4$.

From the bottom row, $a + 10 = b$.

Using $a = 4$, this gives $b = 14$.

[3 marks available — 1 mark for correctly multiplying the matrices to find equations for a and b, 1 mark for the correct value of a and 1 mark for the correct value of b]

2 The transformation maps the point (1, 0) to the point (–1, 0) and the point (1, 1) to the point (–1, 1), so the transformation is a reflection *[1 mark]* in the y-axis *[1 mark]*.

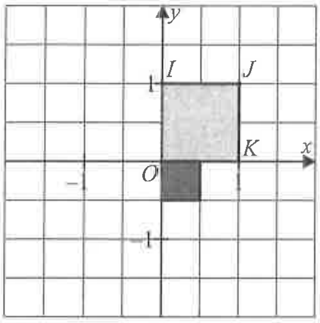
[2 marks available in total — as above]

3 Multiply the two transformation matrices, with the matrix for the first transformation on the right:

$\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}\begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix} = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$

[2 marks available — 1 mark for multiplying the matrices in the correct order, 1 mark for the correct answer]

4 The matrix for the combined transformation is:
$$\begin{pmatrix}0.5 & 0\\0 & 0.5\end{pmatrix}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}=\begin{pmatrix}0.5 & 0\\0 & -0.5\end{pmatrix}$$
Apply this to each point of the unit square:
$$\begin{pmatrix}0.5 & 0\\0 & -0.5\end{pmatrix}\begin{pmatrix}0\\0\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$
$$\begin{pmatrix}0.5 & 0\\0 & -0.5\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}0.5\\0\end{pmatrix}$$
$$\begin{pmatrix}0.5 & 0\\0 & -0.5\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}0\\-0.5\end{pmatrix}$$
$$\begin{pmatrix}0.5 & 0\\0 & -0.5\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}0.5\\-0.5\end{pmatrix}$$



[4 marks available — 1 mark for multiplying the matrices in the correct order, 1 mark for the correct combined matrix, 2 marks if all 4 points are plotted correctly, otherwise 1 mark if 2 or 3 points are plotted correctly]

For this question, you could have worked out that the first transformation matrix represents a reflection in the x-axis, and the second transformation matrix represents an enlargement of scale factor 0.5 and centre (0, 0), then drawn the transformed shape.

Pages 46-47: Geometry

1 Angle $DBC = (180° - 56°) \div 2 = 62°$ ***[1 mark]***
Angle $DBA = 180° - 62° = 118°$ ***[1 mark]***
Angle $BAD = (180° - 118°) \div 2 = 31°$ ***[1 mark]***
[3 marks available in total — as above]

2 Angle $AHD = 77°$ (vertically opposite angles) ***[1 mark]***
Angle $HDC = 180° - 122° = 58°$ (angles on a straight line) ***[1 mark]***
Angle $DCA = 360° - 90° - 77° - 58° = 135°$
(angles in a quadrilateral) ***[1 mark]***
Angle $x = 180° - 135° = 45°$ (angles on a straight line) ***[1 mark]***
[4 marks available in total — as above]

3 Angles in a triangle add up to 180°, so:
$12x + 7x + 17x = 180°$
$36x = 180°$ ***[1 mark]***
$x = 180 \div 36 = 5°$ ***[1 mark]***
$ACB = 7x = 7 \times 5 = 35°$ ***[1 mark]***
[3 marks available in total — as above]

4 Angles EAG and BDE are alternate angles so are equal, and angles on a straight line add up to 180°,
so angle EAG = angle $BDE = 180° - 119° = 61°$ ***[1 mark]***
Angles on a straight line add up to 180°,
so angle $EGA = 180° - 156° = 24°$ ***[1 mark]***
Angles in a triangle add up to 180°,
so angle $AEG = 180° - 61° - 24° = 95°$ ***[1 mark]***
Angles on a straight line add up to 180°,
so angle $x = 180° - 95° = 85°$ ***[1 mark]***
[4 marks available in total — as above]
There are other ways to do this — as long as you show all your working and give reasons for each step (and get the right answer of course), you'll get full marks.

5 a) Angles on a straight line add up to 180°,
so angle $AEC = 180° - 72° = 108°$ ***[1 mark]***
Angles in a quadrilateral add up to 360°,
so angle $m = 360° - 90° - 104° - 108° = 58°$ ***[1 mark]***
[2 marks available in total — as above]

b) Angles in a triangle add up to 180°,
so angle $n = 180° - 90° - 58° = 32°$ ***[1 mark]***

6 Interior angles add up to 180°,
so $(5y + 7)° + (2y + 5)° = 180°$ ***[1 mark]***
$7y = 168$, so $y = 24$ ***[1 mark]***
Interior angles add up to 180°,
so $6x° + (3y - 9)° = 180°$ ***[1 mark]***
Using your value of y,
$6x = 180 + 9 - (3 \times 24) = 117$ ***[1 mark]***
$x = 19.5$ ***[1 mark]***
[5 marks available in total — as above]

Page 48: Area

1 Area of a trapezium $= \frac{1}{2}(a + b) \times h$,
so $23\ 700 = \frac{1}{2}(215 + 180) \times x$ ***[1 mark]***
$23\ 700 = 197.5x$ ***[1 mark]***
$x = 120$ m ***[1 mark]***
[3 marks available in total — as above]

2 a) Area of full circle $= \pi \times 30^2 = 900\pi$ cm²
Area of sector $= (80 \div 360) \times$ area of circle
$= (80 \div 360) \times 900\pi$ cm²
$= 200\pi$ cm²
[2 marks available — 1 mark for a correct method to calculate the area of the sector, 1 mark for the correct answer]
Remember, if you're asked for the exact answer, leave it in terms of π.

b) Circumference of full circle $= \pi \times 30 \times 2 = 60\pi$ cm ***[1 mark]***
Length of arc $= (80 \div 360) \times$ circumference of circle
$= (80 \div 360) \times 60\pi$ cm
$= 41.887...$ cm ***[1 mark]***
Perimeter of sector $= 41.887... + 30 + 30 = 101.887...$
$= 102$ cm (3 s.f.) ***[1 mark]***
[3 marks available in total — as above]

3 Area of $ABC = \frac{1}{2} \times 6a \times 8a = 24a^2$ ***[1 mark]***
$BP = \frac{3}{4} \times 8a = 6a$ and $BN = \frac{1}{2} \times 6a = 3a$
Area of $BPN = \frac{1}{2} \times 3a \times 6a = 9a^2$ ***[1 mark]***
$BM = \frac{1}{2} \times 8a = 4a$ and $BQ = \frac{1}{3} \times 6a = 2a$
Area of $BMQ = \frac{1}{2} \times 2a \times 4a = 4a^2$ ***[1 mark]***
Shaded area $= 24a^2 - 9a^2 + 4a^2$ ***[1 mark]*** $= 19a^2$ ***[1 mark]***
[5 marks available in total — as above]

Page 49: Surface Area and Volume

1 Surface area of curved part of hemisphere =
½ × surface area of a sphere $= ½ \times 4 \times \pi \times 11^2$ ***[1 mark]***
$= 242\pi$ cm² ***[1 mark]***
Surface area of curved part of cone $= \pi \times 8 \times 18$ ***[1 mark]***
$= 144\pi$ cm² ***[1 mark]***
Surface area of flat top of hemisphere $= (\pi \times 11^2) - (\pi \times 8^2)$
$= 57\pi$ cm² ***[1 mark]***
Total surface area $= 242\pi + 144\pi + 57\pi$
$= 443\pi$ cm² ***[1 mark]***
[6 marks available in total — as above]

2 $40\pi = \frac{1}{3} \times \pi r^2 \times 7.5$ ***[1 mark]***
$r^2 = 16$ ***[1 mark]***
$r = 4$ cm ***[1 mark]***
[3 marks available in total — as above]

3 Volume of sphere = $\frac{4}{3} \times \pi \times (6x)^3$ ***[1 mark]***
$= 288x^3\pi$ mm^3 ***[1 mark]***
Volume of one cylinder = $\pi \times x^2 \times 16 = 16x^2\pi$ mm^3 ***[1 mark]***
Total volume of cylinders = $16x^2\pi \times 126 = 2016x^2\pi$ mm^3 ***[1 mark]***
$288x^3\pi = 2016x^2\pi$ ***[1 mark[***
$288x = 2016$
$x = \frac{2016}{288} = 7$ ***[1 mark]***
[6 marks available in total — as above]
Normally, you shouldn't divide by x (in case it's 0), but here it's OK, as you know it can't be 0.

Pages 50-51: Circle Geometry

1 a) Angle $AOC = 360° - 210° = 150°$
Angle $ADC = 150° \div 2 = 75°$ ***[1 mark]***
(Angle at the centre is 2 × angle at circumference.) ***[1 mark]***
[2 marks available in total — as above]

b) Opposite angles in a cyclic quadrilateral sum to 180°. ***[1 mark]***

2 Angle $ACB = 74°$ (angles in the same segment are equal) ***[1 mark]***
Angle $ABC = 90°$ (angle in a semi-circle) ***[1 mark]***
Angle $x = 180° - 90° - 74° = 16°$ ***[1 mark]***
[3 marks available in total — as above]

3 Angle $DBF = 90°$ (angle in a semi-circle) ***[1 mark]***
Angle $BDF = 180° - 90° - 62° = 28°$ ***[1 mark]***
Angle $BOD = 180° - 2 \times 28° = 124°$ (BOD is an isosceles triangle) ***[1 mark]***
Angle ABO = angle $ADO = 90°$ (tangent and a radius meet at 90°) ***[1 mark]***
$x = 360° - 90° - 90° - 124° = 56°$ (angles in a quadrilateral add up to 360°) ***[1 mark]***
[5 marks available in total — as above]

4 a) Angle $OPS = 90°$ (tangent and a radius meet at 90°) ***[1 mark]***
Angle $QPR = 90° - 66° = 24°$ ***[1 mark]***
[2 marks available in total — as above]

b) Alternate segment theorem ***[1 mark]***
The angle between a chord and a tangent is equal to the angle made at the circumference of the circle by two lines drawn from the ends of the chord.

5 By the alternate segment theorem, angle EDA = angle DCA, so $x = 58°$ ***[1 mark]***. By the alternate segment theorem again, angle EDA = angle $DBA = 58°$ ***[1 mark]***. Opposite angles in a cyclic quadrilateral sum to 180°, so angle ADC + angle $ABC = 180°$ ***[1 mark]***. So $87° + 58° + y = 180°$, so $y = 35°$ ***[1 mark]***.
[4 marks available in total — as above]
If you'd used a different method for any of the questions in this topic, you'll still get the marks (as long as you explain each step and get the right answer).

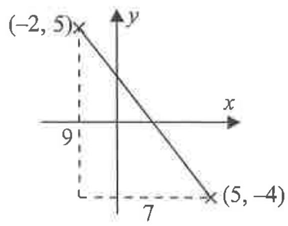
Section Five — Pythagoras and Trigonometry

Page 52: Pythagoras' Theorem

1 $AB^2 = 11^2 + 15^2$ ***[1 mark]***
$AB = \sqrt{121 + 225} = \sqrt{346}$
$AB = 18.6$ mm (3 s.f.) ***[1 mark]***
[2 marks available in total — as above]

2 Let h be the height of the triangle. Then, by splitting the triangle into two and using the Pythagorean triple 8, 15, 17, $h = 15$ m ***[1 mark]***
Area $(A) = \frac{1}{2} \times$ base × height
$A = \frac{1}{2} \times 16 \times 15$
$A = 120$ m^2 ***[1 mark]***
[2 marks available in total — as above]

3 $AD = AB = 12.1$ cm, $DC = BC = 23.4$ cm
Length of EA:
$12.1^2 = 8^2 + EA^2$ ***[1 mark]***
$EA = \sqrt{146.41 - 64}$
$EA = 9.077...$ ***[1 mark]***
Length of CE:
$23.4^2 = 8^2 + CE^2$ ***[1 mark]***
$CE = \sqrt{547.56 - 64}$
$CE = 21.989...$ ***[1 mark]***
Perimeter = $12.1 + 23.4 + EA + CE = 66.57$ cm (2 d.p.) ***[1 mark]***
[5 marks available in total — as above]

4 

Difference in x-coordinates = $5 - -2 = 7$
Difference in y-coordinates = $5 - -4 = 9$
Distance between points = $\sqrt{7^2 + 9^2}$ ***[1 mark]***
= 11.4 units (3 s.f.) ***[1 mark]***
[2 marks available in total — as above]

Page 53: Trigonometry — Sin, Cos, Tan

1 Split ABC into two right-angled triangles, and let $x = AC \div 2$. Then
$\cos 48° = \frac{x}{9.5}$ ***[1 mark]***
$x = 9.5 \times \cos 48°$ ***[1 mark]***
$x = 6.356...$
$AC = 6.356... \times 2 = 12.71$ cm (2 d.p) ***[1 mark]***
[3 marks available in total — as above]

2 $\tan y = \frac{AC}{12} \Rightarrow 0.75 = \frac{AC}{12}$, ***[1 mark]***
so $AC = 0.75 \times 12 = 9$ cm ***[1 mark]***
$\sin x = \frac{AD}{9}, \frac{2}{3} = \frac{AD}{9}$, ***[1 mark]*** so $AD = \frac{2}{3} \times 9$
$AD = 6$ cm ***[1 mark]***
[4 marks available in total — as above]

3 a) $\tan HGM = \frac{7}{11}$ ***[1 mark]***
$HGM = \tan^{-1}\left(\frac{7}{11}\right)$ ***[1 mark]***
$HGM = 32.5°$ (3 s.f.) ***[1 mark]***
[3 marks available in total — as above]

b) EG bisects the angle FEH, so find angle FEM:
$\tan FEM = \frac{7}{4}$ ***[1 mark]***
$FEM = \tan^{-1}\left(\frac{7}{4}\right)$ ***[1 mark]***
$FEM = 60.255...°$
$FEH = 60.255... \times 2 = 121°$ (3 s.f.) ***[1 mark]***
[3 marks available in total — as above]

Pages 54-55: The Sine and Cosine Rules

1 a) $AB^2 = 31^2 + 37^2 - (2 \times 31 \times 37 \times \cos 82°)$ ***[1 mark]***
$AB = \sqrt{2330 - 2294 \times \cos 82°}$ ***[1 mark]***
$AB = 44.8$ cm (3 s.f.) ***[1 mark]***
[3 marks available in total — as above]

b) Area $= \frac{1}{2} \times 31 \times 37 \times \sin 82°$ ***[1 mark]***
Area = 568 cm^2 (3 s.f.) ***[1 mark]***
[2 marks available in total — as above]

2 a) $\frac{BD}{\sin 26°} = \frac{10}{\sin 44°}$ ***[1 mark]***
$BD = \frac{10}{\sin 44°} \times \sin 26°$ ***[1 mark]***
$BD = 6.3106... = 6.31$ m (3 s.f) ***[1 mark]***
[3 marks available in total — as above]

b) $\frac{7}{\sin BDC} = \frac{6.3106...}{\sin 62°}$ ***[1 mark]***
$\sin BDC = \frac{\sin 62°}{6.3106...} \times 7$
$BDC = \sin^{-1}(0.9794...)$ ***[1 mark]***
$BDC = 78.4°$ (3 s.f.) ***[1 mark]***
[3 marks available in total — as above]

3 a) $\sin 35° = \frac{12}{AC}$ ***[1 mark]***
$AC = \frac{12}{\sin 35°}$ ***[1 mark]***
$AC = 20.921... = 20.9$ cm (to 3 s.f.) ***[1 mark]***
[3 marks available in total — as above]

b) Angle $CAD = 90° - 35° = 55°$ ***[1 mark]***
$CD^2 = 20.921...^2 + 46^2 - (2 \times 20.921... \times 46 \times \cos 55°)$ ***[1 mark]***
$CD = \sqrt{1449.703...}$
$CD = 38.074...$ cm ***[1 mark]***
Perimeter of $ACD = 20.921... + 38.074... + 46$
$= 105$ cm (to the nearest cm) ***[1 mark]***
[4 marks available in total — as above]

4 Find one angle using the cosine rule. Using angle A:
$\cos A = \frac{8^2 + 5^2 - 11^2}{2 \times 8 \times 5}$ ***[1 mark]***
$A = \cos^{-1}\left(\frac{-2}{5}\right)$ ***[1 mark]***
$A = 113.578...°$ ***[1 mark]***
Area $= \frac{1}{2} \times 8 \times 5 \times \sin 113.578...°$ ***[1 mark]***
Area $= 18.3303... = 18.3$ cm^2 (to 3 s.f.) ***[1 mark]***
[5 marks available in total — as above]

5 First, split $ABCD$ into two triangles, ABC and ACD.
$\frac{32}{\sin ACB} = \frac{49}{\sin 103°}$ ***[1 mark]***
$\sin ACB = \frac{\sin 103°}{49} \times 32$ ***[1 mark]***
Angle $ACB = \sin^{-1}(0.636...)$
$ACB = 39.518...°$ ***[1 mark]***
Angle $BAC = 180° - 103° - 39.518...° = 37.481...°$ so,
Area of $ABC = \frac{1}{2} \times 49 \times 32 \times \sin 37.481...°$ ***[1 mark]***
Area of $ABC = 477.071...$cm^2 ***[1 mark]***
Angle $ACD = 81° - 39.518...° = 41.481...°$ so
Area of $ACD = \frac{1}{2} \times 49 \times 43 \times \sin 41.481...°$ ***[1 mark]***
Area of $ACD = 697.819...$cm^2 ***[1 mark]***
Area of $ABCD = 477.071... + 697.819... = 1170$ cm^2 ***[1 mark]***
[8 marks available in total — as above]

Page 56: 3D Pythagoras

1 $BH^2 = 8^2 + 4^2 + 5^2$ ***[1 mark]***
$BH = \sqrt{105}$
$BH = 10.2$ cm (3 s.f.) ***[1 mark]***
[2 marks available in total — as above]

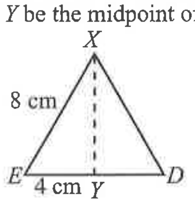
2 Using Pythagoras' Theorem on triangle AXV:
$AX^2 = 8.9^2 - 7.2^2 = 27.37$, so $AX = \sqrt{27.37}$ ***[1 mark]***
and $AC = 2\sqrt{27.37}$ ***[1 mark]***
Now using Pythagoras' Theorem on triangle ABC:
$AB^2 = (2\sqrt{27.37})^2 - 4.2^2 = 91.84$ ***[1 mark]***,
so $AB = \sqrt{91.84} = 9.58$ cm (3 s.f.) ***[1 mark]***
[4 marks available in total — as above]

Page 57: 3D Trigonometry

1 a) $FD^2 = 3^2 + 4^2 + 9^2$ ***[1 mark]***
$FD = \sqrt{106}$
$FD = 10.3$ cm (3 s.f.) ***[1 mark]***
[2 marks available in total — as above]

b) $\sin FDG = \frac{4}{\sqrt{106}}$ ***[1 mark]***
$FDG = \sin^{-1}\left(\frac{4}{\sqrt{106}}\right)$
$FDG = 22.9°$ (1 d.p.) ***[1 mark]***
[2 marks available in total — as above]

2 $XE = 8$ cm as the hexagon is made from equilateral triangles.
Let Y be the midpoint of ED:



$XY^2 = 8^2 - 4^2 = 48$, so $XY = \sqrt{48}$ cm ***[1 mark]***
The angle between planes VED and $ABCDEF$ is angle VYX.
$\tan VYX = \frac{15}{\sqrt{48}}$ ***[1 mark]***
$VYX = \tan^{-1}\left(\frac{15}{\sqrt{48}}\right)$
$VYX = 65.2°$ (1 d.p.) ***[1 mark]***
[3 marks available in total — as above]

Page 58: Trig Values

1 $\tan 45° = 1$
$\sin 30° = 0.5$
$\cos 30° = \frac{\sqrt{3}}{2}$
$\tan 60° = \sqrt{3}$
[2 marks available — 2 marks for all four values correct, otherwise 1 mark for two or three values correct]

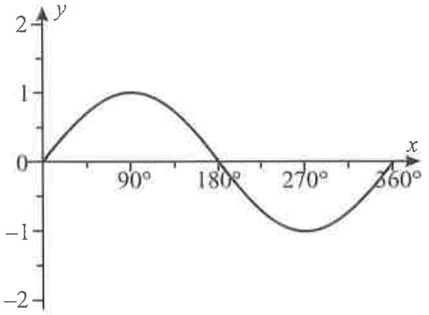
2 From triangle ABC, $\sin 60° = \frac{AC}{\sqrt{27}}$ ***[1 mark]***
So $AC = \sqrt{27} \sin 60° = \sqrt{27} \times \frac{\sqrt{3}}{2}$ ***[1 mark]***
$= \frac{\sqrt{81}}{2} = \frac{9}{2}$ cm ***[1 mark]***
Angle $ACD = 180° - 90° - 45° = 45°$, so triangle ACD is isosceles,
so $AD = AC = \frac{9}{2}$ cm ***[1 mark]***.
[4 marks available in total — as above]

3 $XZ^2 = 1^2 + 1^2 = 2$, so $XZ = \sqrt{2}$ cm ***[1 mark]***
Angle $XZY = 45°$ as the triangle is isosceles ***[1 mark]***
$\cos 45° = \frac{A}{H} = \frac{1}{\sqrt{2}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2}$ ***[1 mark]***
[3 marks available in total — as above]
Here you had to rationalise the denominator to get the answer in the correct form — by multiplying top and bottom by $\sqrt{2}$.

Pages 59-60: Solving Trig Equations in a Given Interval

1 $x = 230°$ (accept $228° - 232°$) *[1 mark]*

2 a)



[2 marks available — 2 marks for the correct graph, otherwise 1 mark for a graph of the correct shape but passing through the wrong points]

b) $-1 \leq f(x) \leq 1$ *[1 mark]*

c) $f(x) = 0.25$ has 2 solutions
$f(x) = 1$ has 1 solution
$f(x) = 0$ has 3 solutions
$f(x) = -2$ has 0 solutions
[2 marks available — 2 marks for all 4 correct, otherwise 1 mark for 2 or 3 correct]

3 Using the symmetry of the graph, $x = 180° - 18° = 162°$ *[1 mark]*
and $x = 360° - 18° = 342°$ *[1 mark]*
[2 marks available in total — as above]

4 $x = \cos^{-1}(0.65) = 49.458...° = 49.5°$ (1 d.p.) *[1 mark]*
Looking at the graph of cos x, you can see that cos x has a line of symmetry along the line $x = 180°$, so there's another solution at $x = 360° - 49.458...°$ *[1 mark]*
$= 310.541...° = 310.5°$ (1 d.p.) *[1 mark]*
[3 marks available in total — as above]
When you're solving a trig equation, you should sketch the relevant trig graph to make sure you include all the solutions in the given interval.

5 Rearrange the equation to get sin x on its own:
$5 \sin^2 x = 2$
$\sin^2 x = \frac{2}{5} = 0.4$
$\sin x = \pm\sqrt{0.4}$ (= 0.632..., −0.632...) *[1 mark]*
If $\sin x = 0.632...$, $x = 39.231...° = 39.2°$ (1 d.p.) *[1 mark]*,
and from the shape of the graph,
$x = 180° - 39.231...° = 140.768...° = 140.8°$ (1 d.p.) *[1 mark]*
If $\sin x = -0.632...$, $x = -39.231...°$, which isn't in the given interval, so use the sin graph to find the solutions which are:
$x = 180° + 39.231...° = 219.231...° = 219.2°$ (1 d.p.) *[1 mark]*
and $x = 360° - 39.231...° = 320.768...° = 320.8°$ (1 d.p.) *[1 mark]*
[5 marks available in total — as above]

Page 61: Trig Identities

1 $2 - \sin x = 2\cos^2 x$, and $\cos^2 x \equiv 1 - \sin^2 x$
$2 - \sin x = 2(1 - \sin^2 x)$
$2 - \sin x = 2 - 2\sin^2 x$
$2\sin^2 x - \sin x = 0$
Now factorise:
$\sin x(2\sin x - 1) = 0$, so $\sin x = 0$ or $\sin x = \frac{1}{2}$.
For $\sin x = 0$, $x = 0°$, $180°$ and $360°$.
For $\sin x = \frac{1}{2}$, $x = 30°$ and (using the shape of the graph)
$180° - 30° = 150°$.
[6 marks available — 1 mark for correct substitution using trig identity, 1 mark for factorising quadratic in sin x, 1 mark for finding correct values of sin x, 3 marks for all five solutions, otherwise 2 marks for three or four solutions, otherwise 1 mark for two solutions]

2 a) $2(1 - \cos x) = 3\sin^2 x$, and $\sin^2 x \equiv 1 - \cos^2 x$
$2(1 - \cos x) = 3(1 - \cos^2 x)$
$2 - 2\cos x = 3 - 3\cos^2 x$
$3\cos^2 x - 2\cos x - 1 = 0$
[2 marks available — 1 mark for substituting in $\sin^2 x \equiv 1 - \cos^2 x$, 1 mark for reaching the correct answer]

b) $3\cos^2 x - 2\cos x - 1 = 0$
$(3\cos x + 1)(\cos x - 1) = 0$ *[1 mark]*
$\cos x = -\frac{1}{3}$ or $\cos x = 1$ *[1 mark]*
For $\cos x = 1$, $x = 0°$ and $360°$
For $\cos x = -\frac{1}{3}$, $x = 109.4712...° = 109.5°$ (1 d.p.)
and (using the shape of the graph)
$x = 360° - 109.4712...° = 250.5287...° = 250.5°$ (1 d.p.)
[2 marks for all four correct solutions, otherwise 1 mark for two or three correct solutions]
[4 marks available in total — as above]

3 $\sin^2\theta + \cos^2\theta \equiv 1$, so
$\sin^2\theta\tan\theta \equiv (1 - \cos^2\theta)\tan\theta$ *[1 mark]*
$\equiv \tan\theta - \cos^2\theta\tan\theta$
$\equiv \tan\theta - \cos^2\theta\frac{\sin\theta}{\cos\theta}$ *[1 mark]*
$\equiv \tan\theta - \sin\theta\cos\theta$ as required *[1 mark]*
[3 marks available in total — as above]

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